

## Foreword

By Dr. Wayne G. Sanstead,  
State Superintendent

There is perhaps no greater responsibility for a community than to provide for the care and education of its citizens. We stand together, committed to the advancement of quality education for all our students. It is toward this aim that I am pleased to issue this document, the *North Dakota Mathematics Content and Achievement Standards*. This document represents an important step in defining and implementing what constitutes a quality education for North Dakota citizens.

### *The State's Protocols for Developing Standards*

As a matter of public policy, the North Dakota Department of Public Instruction believes that public education stakeholders must define "what students should know and be able to do." State law (NDCC 15.1-04-03) places responsibility for the writing of state academic standards with the State Superintendent. State law (NDCC 15.1-04-04; 15.1-21-08) also places responsibility with the State Superintendent for the supervision of State assessments that are based on the State's academic standards. To this end, the Department of Public Instruction has worked closely with the State's educators, through a clearly articulated process, to develop academic standards and aligned assessments that reflect the profession's best insights into what constitutes a quality education for every citizen.

The North Dakota State content and achievement standards offer guidance in core curriculum areas while, at the same time, they allow for, indeed *encourage*, a dynamic and living curriculum created at the local school district level. To ensure educational relevance, the North Dakota State content and achievement standards are (1) based on academic standards developed nationally by various professional education associations, (2) periodically revised as suggested by classroom and community experiences and expectations, and (3) widely supported by state and national education policymakers.

While the North Dakota State content and achievement standards represent an official, statewide reference point for content and proficiency, local school districts are encouraged to use the State's content and achievement standards as guides in the development of local, customized curriculum in

the core content areas. Put another way, standards-based education requires that a community agree upon what skills and abilities students should have upon leaving high school and what a worthwhile K-12 educational experience should look like in the classroom.

The Department seeks to engage stakeholders in the development and performance reporting of a valid and reliable educational accountability system. To ensure that the State's accountability system engenders confidence among constituents, the Department has established a system of prescribed activities that are designed to assure procedural validity and reliability, product quality, and systemic integrity. The Department, with the assistance of professional educators from across the State, has established process and content protocols to articulate the governing rules for the development of State content and achievement standards and assessments.

I encourage all citizens to familiarize themselves with the process used to define, review, and implement the State's challenging content and achievement standards. The *North Dakota Standards and Assessment Development Protocols* (refer to the following website: <http://www.dpi.state.nd.us/standard/protocols.pdf>) identify the procedures Department staff and statewide educator design teams follow regarding the staffing, design process, formatting, and content of all documents. These protocols ensure that quality assurance measures are observed and that the process and its resulting product are valid. The *North Dakota Mathematics Content and Achievement Standards* document has been developed with care and attention to the requirements of the State standards protocols.

### *State's Accountability for Every Student*

Article VIII of the State's Constitution places a high-level responsibility on the State to ensure the literacy of every citizen. A high-quality education is the right of every student.

Assuring comparable educational opportunity is a primary responsibility of the State's education system. The State's challenging content and

achievement standards define what students should know and be able to do. In a sense, these standards represent a state contract with our students.

I urge school districts to build their respective curricula upon these worthy standards. A school's curriculum encompasses that collection of textbooks, media, experiences, and instruction that guide a student's exposure to the standards. The standards define the "what" and the curriculum defines the "how" of education.

To hold itself accountable for the educational services it provides through its schools, the State has developed an assessment system that is designed to measure student performance in terms of these State standards. By measuring student achievement in terms of the State's challenging standards, we are able to monitor growth, address deficiencies, and ensure comparability of educational opportunity statewide.

The State's academic assessments are aligned to the State's challenging content and achievement standards. The State measures, through a sampling of test questions, the annual performance of students statewide. Results are analyzed based on overall student achievement and on the performance of student subgroups, i.e., students of different ethnic backgrounds, limited English proficient students, economically disadvantaged students, and students with disabilities.

The State issues annual reports on the progress students make toward overall proficiency in terms of the State's standards. I invite all educators and citizens to learn more about the State's Accountability Plan which details this annual report. (Refer to the following website: <http://www.dpi.state.nd.us/grants/acctwb.pdf>.)

I assure you that our State's accountability system exists to protect the interests of every student—every student.

#### *Continuing Tradition of Improvement and Excellence*

The legacy of the North Dakota educational system is represented by the quality of the students it graduates every year. Every student who graduates at the proficient level from a North Dakota school testifies to the strength of the families, communities, and schools that nurtured and educated that student.

Yet, as long as there are students who graduate from a North Dakota school scoring at levels less than proficient—or who do not graduate at all, then

evidence exists of our need to improve. School improvement requires a healthy network of families, communities, and schools, working together, to achieve the ultimate aim set forth within the State Constitution.

The State standards, by their very nature, define the measure of success we seek to achieve. We cannot summarily claim success as long as any of our students fall below the proficient level as defined by our achievement standards. Proficiency of every student is our mission. This is why we do what we do. Each and every student, in every subgroup, is that important.

North Dakota schools embody a long-standing tradition to build on success and improve where necessary. These standards establish our measures for success. These standards anchor us and guide us. If we, indeed, are to continue to improve as an educational system, then it is these standards that will lead us ultimately to our goal. The North Dakota content and achievement standards are that important to us all.

#### *Gratitude to a Dedicated Profession*

We stand on the shoulders of those educators who have preceded us. Whatever measure of success we have experienced to date rests in large measure on their efforts and dedication. Each passing generation stands as a testament to the spirit of excellence that exists within the education community. Each generation builds upon the strengths of its predecessors in order to secure a better future for those who follow.

The work of developing and implementing the State mathematics standards finds its origins in many past efforts. The contributions of countless educators are astounding and inspiring. It is now for our generation to harvest the best of the past, to incorporate our best insights based on current research, and to restate our mission for future learners.

I wish to recognize each individual team member and the hundreds of reactors who contributed to the writing of this important document. We must be mindful of the many weeks and iterations of background research, discussions, drafting, reflective analysis, debating, and ultimate resolution that have been invested in this document. No words of gratitude can ever do justice to the quality of work or the commitment found within this document.

Now, the work rests with us. Our current and future students depend on us. Let us go forth and touch the future together.



## INTRODUCTION

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. To be a responsible and productive member of today's society a student needs to have a broad, connected and useful knowledge of mathematics.

The *North Dakota Mathematics Content and Achievement Standards* document provides a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on its predecessor, the 1999 *North Dakota Mathematics Content Standards*, this document includes standards and benchmarks and defines levels of achievement at each grade level. In addition to referencing previous state content standards, these current state standards reference the National Council of Teachers of Mathematics', *Principles and Standards for School Mathematics* as well as standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are aligned to these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. "All students" include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who have high intellectual ability; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students,

who, after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Grade level expectations are identified for every classroom, kindergarten through senior high school. These statements reflect what every student should know and be able to do at the end of each specified grade level.

### *Definitions and Document Components*

The *North Dakota Mathematics Content and Achievement Standards* document contains the following organizational components:

- **Content standard:** A description of what students should know and be able to do within mathematics.
- **Topic:** A category within a content standard that associates or aids in the organization of related benchmark expectations and that may carry across grade levels.
- **Grade-level benchmark expectation:** A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, "i.e." means "these specific items," and "e.g." means "for example." Grade level expectations are benchmarked to indicate a higher level of knowledge and skills as the student progresses through the curriculum.
- **Achievement standard:** A description of what a student knows and can do to demonstrate a level of achievement on

a content standard. Descriptors for achievement are set at four levels and are defined as follows:

- **Advanced Proficient.** Demonstrates exemplary understanding or skill and exceeds expected level of performance.
- **Proficient:** Demonstrates understanding or skill and meets expected level of performance.
- **Partially Proficient:** Demonstrates an emerging or developing level of understanding and performance.
- **Novice:** Attempt made; however, lack of understanding and performance is evident.

#### *State Content Standards Format*

Each content standard is presented according the following format.

- **Heading.** A standard is introduced by an overall page heading that identifies the standard's number within the subject and a short descriptive title (e.g., "*Standard 1: Number and Operation*").
- **Content Standards Description.** The standard is defined succinctly in terms of students' expected knowledge or skill (e.g., "*Students understand and use basic and advanced concepts of number and number systems*").
- **Numbering.** Each standard conforms to the following prescribed numbering system.
  1. **Standard.** A standard is identified uniquely by a prescribed two-digit nomenclature (e.g., "8.8"). The first digit refers to the grade level (e.g., *grade 8*). The second digit refers to the standard's listing within the subject (e.g., *the eighth standard*).
  2. **Grade-level benchmark expectation.** A benchmark expectation is identified uniquely by a prescribed three-digit

nomenclature (e.g., "8.8.2"). The first digit refers to the grade level (e.g., *grade 8*). The second digit refers to the standard's listing within the subject (e.g., *the eighth standard*). The third digit refers to the benchmark's listing within the standard (e.g., *the second benchmark within the eighth standard*).

3. **Topics.** Since topics only organize benchmark expectations within a standard and identify no specific knowledge or skill, topics will carry no uniquely identifiable number.

#### *State Achievement Standards Format*

State achievement standards have been developed for all content standards. An achievement standard is a description of what a student knows and can do to demonstrate a level of achievement on a content standard.

Achievement standards guide one's interpretation regarding "how well a student demonstrates knowledge or skill within a content standard." As such, achievement standards aid in defining performance and in establishing "grading" parameters. Achievement standards identify four categories or levels of student achievement: (1) *novice*, (2) *partially proficient*, (3) *proficient*, and (4) *advanced*. The *proficient* level represents how well a student should demonstrate achievement within mathematics at a particular grade level.

All State assessments will report overall student achievement, school achievement, district achievement, and statewide achievement in terms of the four achievement levels.

The State achievement descriptors express the characteristics of each of the four achievement levels for all content standards. In many instances, achievement descriptors are presented for individual benchmark expectations. In some instances, where benchmark expectations show a closer association to each other, achievement descriptors are presented for the entire topic.

To develop an achievement standard, the writing team first identified the focus of student performance for that benchmark. The focus of performance reflects the nature of the benchmark. A focus of performance might be related to the degree of error in the performance, the speed or fluency of the performance, the variety of examples provided, the quality of the performance, the significance of details provided, or the consistency of the performance. For example, the focus of performance for benchmarks that require students to explain a concept, principle, or generalization generally might be expressed through the significance of details or the degree of error. For consistency, writers used a limited set of descriptors for each focus to describe the levels of performance (i.e., advanced, proficient, partially proficient, novice). For example, for the “quality” focus, descriptors for the four levels included insightful, relevant, obvious, and irrelevant, respectively.

Teachers in a school or district should come to consensus on the meaning of these terms, perhaps through professional dialogue and examination of student work. Teachers also will need to help students understand what is meant by these terms by providing examples of student work at each performance level.

#### *State Standards and Local Curriculum Development*

State content standards broadly define what a student should know and be able to do. State content standards become the basis upon which local school districts define their local curriculum. School districts choose those instructional materials and practices that will ensure a rich mathematics curriculum for all students. And clear content standards define all that will be assessed at a grade level. Quality education begins with and springs from challenging content standards.

Mathematics instruction should reflect what both educational research and best practice reveal about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets,

calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs.

However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines. Readers should reference the State’s *Library and Technology Content Standards*, among other sources, in order to ensure a well-rounded mathematics curriculum.

#### *Integrating Mathematics Standards Across the Curriculum*

Individuals encounter mathematics in a wide variety of settings in daily living. Mathematical thinking engages the physical sciences, the social sciences, the arts, language arts, technology, and most other disciplines. Accordingly, the instruction of mathematics should be integrated throughout the curriculum and not be restricted to the instruction that occurs during the confines of a mathematics course. Mathematics constitutes a foundational skill and carries content that touches on a wide variety of other disciplines.

In the development of a school’s overall curriculum, attention should be given to ensure that content linkages are designed across disciplines. Students will optimize their learning whenever direct connections are made within the wider curriculum.

#### *Mathematics As Content and Process*

The notion of what it means to be mathematically literate has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Today, mathematical literacy includes both the processes of mathematics and the content of mathematics. The processes of mathematics include problem solving, mathematical reasoning and proof,

representation and communication of mathematical ideas, and making connections across the various areas of mathematics and between mathematics and other disciplines. These processes are embedded within the content benchmarks in the *North Dakota Mathematics Content and Achievement Standards* to emphasize that mathematics content and mathematics processes should be taught in tandem and by design. This means that, in addition to teaching content, teachers should also teach students the knowledge and skills that are part of each of the processes. Table A illustrates the critical mathematical processes for learning and instruction.

Districts should use the *North Dakota Mathematics Content and Achievement Standards* to design mathematics programs that provide students with opportunities to spend time on a regular basis solving real-world problems and generating, discussing, and writing about mathematical ideas. By doing so, districts will help students develop the mathematical literacy they need to meet the challenges of the twenty-first century.

#### *Personalizing Education: Differentiated Instruction and Alternate Assessment*

All students are to be taught to the State's challenging standards. This is a fundamental principle to ensure that every student will be offered a comparable and equitable opportunity for a quality education. In the course of instruction, it is appropriate to personalize or differentiate instruction for some students based on their individual programming. All students should be introduced to the content, including the language or vocabulary inherent in the standards. Students should be allowed to explore new or unique expressions of the standards that better allow them access to the breadth and depth of the standards. Students should be encouraged to internalize and rearticulate the standards in a manner that advances each student's appreciation, integration, and generalization of the standards' meaning. Students should understand that their education is an ongoing, rich exploration and incorporation of the standards.

In the course of instruction of students with significant cognitive disabilities, it will be appropriate, indeed necessary, to personalize or differentiate instruction to meet their unique needs. Teachers, other support staff, and parents of a student with a significant cognitive disability may, within the context of an individualized education program (IEP) team, determine it necessary to interpret a standard to meet the needs of the learner. Educators might consider what a given grade level benchmark expectation would look like for a given student with a significant disability. Most highly qualified special educators will realize that patterns and algebra, for example, can be taught, but with different teaching strategies and modified expectations. Any such instructional strategies and modified expectations should be referenced within the student's individualized education program. The North Dakota Alternate Assessment for students with significant cognitive disabilities will accept and incorporate appropriate expectation modifications as an element of the student's standards-based Alternate Assessment.

As a matter of policy, no State content or achievement standards document may develop, reference, or otherwise encourage modified benchmark expectations that alter, in any manner, the breadth or depth of the State's challenging standards. The differentiation of any instruction is limited solely to a local individualized education program team and allowed only for students with significant cognitive disabilities. Educators and parents are encouraged to consult the State's guidance on the development of a student's individualized education program. Refer to the following website for additional information concerning the development of individualized education programs: <http://www.dpi.state.nd.us/speced/guide/iep/index.shtm>

**Table A**  
**Critical Mathematical Processes for Learning and Instruction**

<b>Mathematics Processes</b>	<b>Importance of the Process</b>	<b>What Students Can Do</b>	<b>Critical Instructional Components</b>
<b>Problem Solving</b>	<ul style="list-style-type: none"> <li>▪ Develops flexibility, perseverance, and strategies for dealing with unfamiliar situations</li> <li>▪ Is a major part of learning and “doing” mathematics</li> </ul>	<ul style="list-style-type: none"> <li>▪ Find ways to solve problems when no routine path is apparent</li> <li>▪ Use a variety of strategies and technology to solve meaningful mathematical problems</li> </ul>	<ul style="list-style-type: none"> <li>▪ Frequent and regular opportunities to solve non-routine problems, including those problems students pose themselves</li> <li>▪ Many opportunities to use technology (electronic devices as well as hands-on manipulatives) to explore, perform, and organize mathematical strategies and processes</li> </ul>
<b>Mathematical Reasoning and Proof</b>	<ul style="list-style-type: none"> <li>▪ Helps students understand that mathematics makes sense</li> <li>▪ Develops ability to identify patterns or regularities in mathematical and real-world situations</li> </ul>	<ul style="list-style-type: none"> <li>▪ Gather data, make conjectures, assemble evidence, and build arguments to support or refute conjectures</li> </ul>	<ul style="list-style-type: none"> <li>▪ Opportunities to acquire mathematics facts and skills</li> <li>▪ Opportunities to express and interpret math ideas and relationships</li> <li>▪ Situations that require inductive and deductive reasoning skills</li> </ul>
<b>Mathematical Communication</b>	<ul style="list-style-type: none"> <li>▪ Prepares students to use mathematics effectively and efficiently</li> <li>▪ Helps students clarify, refine, and consolidate their thinking</li> </ul>	<ul style="list-style-type: none"> <li>▪ Present mathematical ideas in written, visual, and oral formats</li> </ul>	<ul style="list-style-type: none"> <li>▪ Opportunities to read, write, and discuss ideas in problem situations</li> <li>▪ Opportunities to think and talk about mathematics by asking questions and working with others</li> </ul>
<b>Mathematical Representation</b>	<ul style="list-style-type: none"> <li>▪ Expands students’ ability to model and interpret real-world and mathematical phenomena</li> </ul>	<ul style="list-style-type: none"> <li>▪ Translate information from real world into mathematical language using signs, symbols, graphs, and terms of mathematics</li> <li>▪ Represent mathematical ideas and conclusions through graphs, charts, formulas, physical objects, drawings</li> </ul>	<ul style="list-style-type: none"> <li>▪ Opportunities to create and use a variety of ways to represent mathematical ideas</li> </ul>
<b>Mathematical Connections</b>	<ul style="list-style-type: none"> <li>▪ Makes mathematics meaningful and useful</li> <li>▪ Develops flexibility in career options</li> </ul>	<ul style="list-style-type: none"> <li>▪ Make links among mathematical ideas and to other disciplines</li> <li>▪ Portray mathematics as an integrated whole that permeates life inside and outside of school</li> </ul>	<ul style="list-style-type: none"> <li>▪ Situations that require application of mathematical facts, skills, concepts, and processes to problems encountered in life</li> </ul>