

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

Content standard – A description of what students should know and be able to do within a particular content discipline or subject.

Subtopic - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

Benchmark expectation - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

Achievement standard A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

Advanced Proficient -Demonstrates exemplary understanding and exceeds expected level of performance

Proficient - Demonstrates understanding and meets expected level of performance.

Partially Proficient - Demonstrates an emerging or developing level of understanding and performance.

Novice – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 1</p> <p>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</p> <ul style="list-style-type: none"> • Count and order numbers to 100 • Identify and write numerals to 100 • Count backward from 20 • Count by 2's, 5's, and 10's • Group objects by 2's, 5's, and 10's • Identify position using ordinal numbers 	<p>Students correctly count and order numbers, identify and write numerals beyond 100.</p> <p>Students correctly count backward from 100.</p> <p>Students correctly skip count and group objects by 2's, 5's, 10s and other numbers.</p> <p>Students correctly identify position using ordinal numbers.</p>	<p>Students correctly count and order numbers, identify and write numerals to 100.</p> <p>Students correctly count backward from 20.</p> <p>Students correctly count by 2's, 5's, and 10's.</p> <p>Students correctly group objects by 2's, 5's, and 10's.</p> <p>Students correctly identify position using ordinal numbers.</p>	<p>Students count and order numbers, identify and write numerals to 100 with few errors.</p> <p>Students count backward from 20 with few errors.</p> <p>Students correctly count by 2's, 5's, or 10's.</p> <p>Students group objects by 2's, 5's, or 10's with few errors.</p> <p>Students inconsistently identify position using ordinal numbers.</p>	<p>Students incorrectly count and order numbers.</p> <p>Students identify and write numerals to 100 with many errors.</p> <p>Students incorrectly count backward from 20.</p> <p>Students are unable to count by 2's, 5's, or 10's.</p> <p>Students group objects by 2's, 5's, or 10's with many errors.</p> <p>Students show limited knowledge when identifying position using ordinal numbers.</p>

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Connect number words and numerals to the quantities they represent (0 - 10) • Represent and explain fractions (i.e., one half, one fourth) as part of a whole and part of a set using concrete materials/drawings • Identify place value for ones and tens • Compare two digit numbers using symbols; i.e. >, <, = 	Students correctly connect number words and numerals beyond 10 to the quantities they represent.	Students correctly connect number words and numerals to the quantities they represent.	Students connect number words and numerals to the quantities they represent with some errors.	Students are unable to connect number words and numerals to the quantities they represent.
	Students correctly demonstrate fractions as part of a whole and part of a set using concrete materials/drawings beyond grade level expectations.	Students correctly demonstrate fractions as part of a whole and part of a set using concrete materials/drawings.	Students inconsistently demonstrate fractions as part of a whole and part of a set using concrete materials/drawings.	Students incorrectly demonstrate fractions as part of a whole and part of a set using concrete materials/drawings.
	Students correctly identify place value beyond ones and tens.	Students correctly identify place value for ones and tens.	Students identify place value for ones and tens with few errors.	Students show limited knowledge when identifying place value for ones and tens.
	Students correctly compare three digit numbers using symbols.	Students correctly compare two digit numbers using symbols.	Students compare two digit numbers with limited understanding of the symbols.	Students are unable to compare two digit numbers using symbols.

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> Use appropriate terms when communicating about addition and subtraction; i.e., sum, difference 	<p>Students automatically use appropriate terms when communicating about addition and subtraction.</p>	<p>Students consistently use appropriate terms when communicating about addition and subtraction.</p>	<p>Students inconsistently use appropriate terms when communicating about addition and subtraction.</p>	<p>Students show limited understanding of addition and subtraction terms.</p>
<p>OPERATIONS AND THEIR PROPERTIES</p> <ul style="list-style-type: none"> Use symbols to write addition and subtraction number sentences; i.e., +, -, = 	<p>Students always use correct symbols to write addition and subtraction number sentences.</p>	<p>Students consistently use correct symbols to write addition and subtraction number sentences.</p>	<p>Students inconsistently use correct symbols to write addition and subtraction number sentences.</p>	<p>Students rarely use correct symbols to write addition and subtraction number sentences.</p>
<p>COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> Recall addition facts and subtraction facts (0-10) 	<p>Students quickly recall addition and subtraction facts beyond those required by the grade level.</p>	<p>Students quickly recall addition and subtraction facts.</p>	<p>Students correctly recall a limited number of addition and subtraction facts.</p>	<p>Students recall few addition and subtraction facts.</p>
<ul style="list-style-type: none"> Estimate the number of objects and check by counting 	<p>Students make reasonable estimates of objects and use more than one strategy to check the estimates.</p>	<p>Students make reasonable estimates of objects and check by counting.</p>	<p>Students make unreasonable estimates of objects and attempt to check by counting.</p>	<p>Students make unreasonable estimates of objects and make no attempt to check by counting.</p>

Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 1</p> <p>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</p> <ul style="list-style-type: none"> • Identify, compare, draw, and sort two-dimensional figures; i.e., circle, triangle, rectangle, square, oval and diamond • Identify three-dimensional objects; i.e., pyramid, cube, cone, cylinder, sphere <p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> • No expectations at this level <p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> • Identify lines of symmetry in two-dimensional figures 	<p>Students correctly identify, compare, draw, and sort two-dimensional figures beyond grade level expectations.</p> <p>Students correctly identify three-dimensional objects beyond grade level expectations.</p> <p>Students demonstrate a thorough understanding of lines of symmetry.</p>	<p>Students correctly identify, compare, draw, and sort two-dimensional figures.</p> <p>Students correctly identify three-dimensional objects.</p> <p>Students accurately identify lines of symmetry of two-dimensional figures.</p>	<p>Students identify, compare, draw, and sort two-dimensional figures with some errors.</p> <p>Students identify three-dimensional objects with some errors.</p> <p>Students accurately identify lines of symmetry of two-dimensional figures with some errors.</p>	<p>Students are unable to identify, compare, draw, and sort two-dimensional figures.</p> <p>Students are unable to identify three-dimensional objects.</p> <p>Students show a lack of understanding of lines of symmetry of two-dimensional figures.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> • Arrange and describe objects in space by position, direction and proximity; e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of 	<p>Students correctly arrange and describe in detail objects in space by position, direction and proximity.</p>	<p>Students correctly arrange and describe objects in space by position, direction, and proximity.</p>	<p>Students arrange and describe objects in space by position, direction, or proximity with few errors.</p>	<p>Students arrange objects in space by position, direction or proximity with many errors.</p>

Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 1</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> • Identify and display various forms of data in their world using charts and graphs; e.g., tally charts and bar graphs • Read and interpret tally charts and picture graphs as sources of information <p>PROBABILITY</p> <ul style="list-style-type: none"> • No expectations at this level <p>STATISTICAL METHODS</p> <ul style="list-style-type: none"> • No expectations at this level <p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> • No expectations at this level 	<p>Students accurately identify, organize, and display various forms of data using charts and graphs.</p> <p>Students accurately read and interpret tally charts and picture graphs to solve problems.</p>	<p>Students accurately identify and display various forms of data in their world using charts and graphs.</p> <p>Students accurately read and interpret tally charts and picture graphs as sources of information.</p>	<p>Students inconsistently identify and display various forms of data in their world using charts and graphs.</p> <p>Students have a general sense of tally charts and picture graphs as sources of information.</p>	<p>Students are unable to identify and display various forms of data in their world using charts and graphs.</p> <p>Students inaccurately read and interpret tally charts and picture graphs as sources of information.</p>

Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 1</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> • Use the days of the week to show knowledge of yesterday, today, and tomorrow • Tell time to the hour and half-hour using digital and analog clocks • Estimate, and verify by measuring, length, weight, or capacity using nonstandard units • Estimate, and verify by measuring length to the nearest inch, foot, and centimeter 	<p>Students use the days of the week to show extensive knowledge of yesterday, today, and tomorrow.</p> <p>Students correctly tell time to the hour, half-hour and beyond using digital and analog clocks.</p> <p>Students make reasonable estimations, verify by using standard and non-standard units and solve problems by applying information learned.</p>	<p>Students use the days of the week to show knowledge of yesterday, today, and tomorrow.</p> <p>Students correctly tell time to the hour and half-hour using digital and analog clocks.</p> <p>Students make reasonable estimations and verify by using standard and non-standard units.</p>	<p>Students use the days of the week to show limited knowledge of yesterday, today, and tomorrow.</p> <p>Students inconsistently tell time to the hour and half-hour using digital and analog clocks.</p> <p>Students make estimations but have difficulty verifying by using standard and non-standard units.</p>	<p>Students have difficulty using the days of the week to show knowledge of yesterday, today, and tomorrow.</p> <p>Students are unable to tell time to the hour and half-hour using digital and analog clocks.</p> <p>Students make unreasonable estimations and lack the ability to verify by using standard and non-standard units.</p>

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Identify a penny, nickel, dime, and quarter and state its value • Count a like set of pennies, nickels, or dimes to \$1.00 • Demonstrate that different combinations of coins (i.e., pennies, nickels and dimes) can have the same value • Sequence events with respect to time; e.g., yesterday, today, tomorrow, seasons 	Students correctly identify all coins and correctly count mixed sets of coins greater than a \$1.00.	Students correctly identify a penny, nickel, dime, and quarter and state the appropriate value.	Students inconsistently identify a penny, nickel, dime, and quarter and state the appropriate value.	Students are unable to identify or state the value of a penny, nickel, dime, and quarter.
	Students use multiple combinations of coins to demonstrate a given value.	Students correctly demonstrate that different combinations of coins can have the same value.	Students count a like set of pennies, nickels, or dimes to \$1.00, with few errors.	Students count a like set of pennies, nickels, or dimes to \$1.00, with several errors.
	Students compare and accurately sequence events with respect to time demonstrating complexity of thought.	Students correctly demonstrate that different combinations of coins can have the same value.	Students inconsistently demonstrate that different combinations of coins can have the same value.	Students lack the ability to demonstrate that different combinations of coins can have the same value.
	Students compare and accurately sequence events with respect to time demonstrating complexity of thought.	Students compare and accurately sequence events with respect to time.	Students show a beginning knowledge when comparing and sequencing events with respect to time with beginning knowledge.	Students lack the knowledge to compare and sequence events with respect to time.

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Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> • Identify the appropriate tool used to measure length (i.e., ruler), weight (i.e., scale), time (i.e., clock, calendar) and temperature (i.e., thermometer) 	Students correctly and consistently identify and use the appropriate tool to measure length, weight, time and temperature.	Students correctly and consistently identify the appropriate tool used to measure length, weight, time and temperature.	Students inconsistently identify the appropriate tool used to measure length, weight, time and temperature.	Students are unable to identify the appropriate tool used to measure length, weight, time and temperature.

Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 1</p> <p>PATTERNS, RELATIONS, AND FUNCTIONS</p> <ul style="list-style-type: none"> • Identify, sort, and classify objects by two or more attributes • Recognize, extend, create and describe patterns <p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> • Understand the commutative property of addition; e.g., $3+5=5+3$ <p>MATHEMATICAL MODELING</p> <ul style="list-style-type: none"> • Use symbols to write simple number sentences; i.e., $+$, $-$, $=$ 	<p>Students use multiple methods to accurately identify, sort and classify objects by two or more attributes.</p> <p>Students correctly recognize, extend, create, and adequately describe complex patterns.</p> <p>Students understand the commutative property of addition and can apply it to problem solving.</p> <p>Students correctly and automatically use symbols to write simple number sentences.</p>	<p>Students accurately identify, sort and classify objects by two or more attributes.</p> <p>Students correctly recognize, extend, create, and adequately describe patterns.</p> <p>Students understand the commutative property of addition.</p> <p>Students correctly and consistently use symbols to write simple number sentences.</p>	<p>Students inconsistently identify, sort and classify objects by two or more attributes.</p> <p>Students correctly recognize, extend, create, and describe simple patterns.</p> <p>Students have a beginning understanding of the commutative property of addition.</p> <p>Students show beginning knowledge in the use of symbols to write simple number sentences.</p>	<p>Students accurately identify, sort and classify objects by only one attribute.</p> <p>Students have difficulty recognizing, extending, creating, and describing patterns.</p> <p>Students lack understanding of the commutative property of addition.</p> <p>Students lack understanding of how to use symbols to write simple number sentences.</p>

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Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
RATES OF CHANGE <ul style="list-style-type: none"> • No expectations at this level 				