

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

**Content standard** – A description of what students should know and be able to do within a particular content discipline or subject.

**Subtopic** - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

**Benchmark expectation** - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

**Achievement standard** A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

**Advanced Proficient** -Demonstrates exemplary understanding and exceeds expected level of performance

**Proficient** - Demonstrates understanding and meets expected level of performance.

**Partially Proficient** - Demonstrates an emerging or developing level of understanding and performance.

**Novice** – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

## Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 11-12</p> <p><b>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</b></p> <ul style="list-style-type: none"> <li>• Translate between radical expressions and expressions involving rational exponents</li>   <li>• Describe the hierarchal relationships (e.g., explain why real numbers are complex) among subsets of the complex number system, i.e., complex, real, and imaginary</li>   <li>• Use imaginary numbers to express the square root of a negative number</li> </ul>	<p>Students correctly translate between radical expressions and expressions involving rational exponents, using multiple representations when appropriate.</p> <p>Students thoroughly describe hierarchal relationships between subsets of the complex number system.</p> <p>Students use imaginary numbers to accurately express square roots of negative numbers and simplify expressions containing them.</p>	<p>Students correctly translate between radical expressions and expressions involving rational exponents.</p> <p>Students correctly describe hierarchal relationships between subsets of the complex number system.</p> <p>Students use imaginary numbers to accurately express the square root of a negative number.</p>	<p>Students demonstrate some understanding when translating between radical expressions and expressions involving rational exponents.</p> <p>Students partially describe hierarchal relationships between subsets of the complex number system.</p> <p>Students show limited understanding of imaginary numbers.</p>	<p>Students are unable to translate between radical expressions and expressions involving rational exponents.</p> <p>Students are unable to describe hierarchal relationships between subsets of the complex number system.</p> <p>Students show no understanding of imaginary numbers.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
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<ul style="list-style-type: none"> <li>Justify the steps of an algebraic process using the properties of the real number system; e.g., write an algebraic proof</li> </ul>	<p>Students describe an algebraic process in complete detail, using the properties of the real number system.</p>	<p>Students describe an algebraic process in adequate detail, using the properties of the real number system.</p>	<p>Students describe an algebraic process in limited detail, using the properties of the real number system.</p>	<p>Students are unable to describe an algebraic process using the properties of the real number system.</p>
<b>OPERATIONS AND THEIR PROPERTIES</b>				
<ul style="list-style-type: none"> <li>Determine which properties of the real number system hold for matrices; e.g., matrix multiplication is not commutative</li> </ul>	<p>Students correctly determine and justify which properties of the real number system hold for matrices.</p>	<p>Students correctly determine which properties of the real number system hold for matrices.</p>	<p>Students demonstrate partial understanding of which properties of the real number system hold for matrices.</p>	<p>Students demonstrate a lack of understanding about which properties of the real number system hold for matrices.</p>
<ul style="list-style-type: none"> <li>Apply basic properties of exponents and logarithms to rewrite algebraic expressions; i.e., power of a product, power of a power, products and quotients of powers, zero and negative exponents, and log of a product, quotient, or power</li> </ul>	<p>Students correctly apply the basic properties of exponents and logarithms to rewrite algebraic expressions, with no errors.</p>	<p>Students correctly apply the basic properties of exponents and logarithms to rewrite algebraic expressions, with minor errors.</p>	<p>Students correctly apply some basic properties of exponents and logarithms to rewrite algebraic expressions, with minor errors.</p>	<p>Students demonstrate a lack of understanding about the basic properties of exponents and logarithms.</p>
<b>COMPUTATIONAL FLUENCY AND ESTIMATION</b>				
<ul style="list-style-type: none"> <li>Add, subtract, and multiply complex numbers</li> </ul>	<p>Students accurately add, subtract, and multiply complex numbers.</p>	<p>Students add, subtract, and multiply complex numbers, with minor errors.</p>	<p>Students add, subtract, and multiply complex numbers, with significant errors.</p>	<p>Students are unable to add, subtract, and multiply complex numbers.</p>

NORTH DAKOTA  
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Multiply matrices containing no more than three rows or columns without the use of technology</li> </ul>	Students accurately perform matrix multiplication.	Students use the correct process to perform matrix multiplication, with minor computational errors.	Students attempt to perform matrix multiplication, with significant process or computational errors.	Students are unable to perform matrix multiplication.

## Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 11-12</p> <p><b>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</b></p> <ul style="list-style-type: none"> <li>Use trigonometric relationships to determine side lengths and angle measures in triangles; i.e., right triangle trigonometry, Law of Sines, and Law of Cosines</li> </ul> <p><b>COORDINATE GEOMETRY</b></p> <ul style="list-style-type: none"> <li>No further expectations</li> </ul> <p><b>TRANSFORMATION AND SYMMETRY</b></p> <ul style="list-style-type: none"> <li>No further expectations</li> </ul> <p><b>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</b></p> <ul style="list-style-type: none"> <li>No further expectations</li> </ul>	<p>Students correctly use trigonometric relationships to accurately determine side lengths and angle measures in triangles.</p>	<p>Students correctly use trigonometric relationships to determine side lengths and angle measures in triangles, with minor computational errors.</p>	<p>Students attempt to use trigonometric relationships to determine side lengths and angle measures in triangles, with significant errors.</p>	<p>Students demonstrate no understanding of trigonometric relationships.</p>

### Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 11-12</p> <p><b>DATA COLLECTION, DISPLAY, AND INTERPRETATION</b></p> <ul style="list-style-type: none"> <li>Choose, construct, and interpret an appropriate display to represent a set of data</li> </ul> <p><b>PROBABILITY</b></p> <ul style="list-style-type: none"> <li>Make predictions based on theoretical probabilities and experimental results</li> </ul> <p><b>STATISTICAL METHODS</b></p> <ul style="list-style-type: none"> <li>Select, calculate, and use appropriate measures of central tendency and spread (i.e., mean, median, mode, range, and quartiles) to draw meaningful conclusions about a set of data</li> </ul>	<p>Students demonstrate a superior level of understanding in their selection, construction, and interpretation of a display for a data set.</p> <p>Students make reasonable predictions based on theoretical probabilities and experimental results and provide insightful analysis.</p> <p>Students select and defend the appropriate measures of central tendency and spread, calculate their values with few computational errors allowed, and use these values to draw meaningful conclusions about a set of data.</p>	<p>Students demonstrate a substantial level of understanding in their selection, construction, and interpretation of a display for a data set.</p> <p>Students make reasonable predictions based on theoretical probabilities and experimental results.</p> <p>Students select appropriate measures of central tendency and spread, calculate their values with few computational errors allowed, and use these values to draw meaningful conclusions about a set of data.</p>	<p>Students demonstrate a general level of understanding in their selection, construction, and interpretation of a display for a data set.</p> <p>Students make weak predictions based on theoretical probabilities and experimental results.</p> <p>Students select measures of central tendency and spread, calculate their values with some errors allowed, and use these values to draw conclusions about a set of data.</p>	<p>Students demonstrate a limited level of understanding in their selection, construction, and interpretation of a display for a data set.</p> <p>Students make unreasonable predictions.</p> <p>Students are unable to select, calculate, or use measures of central tendency to draw conclusions about a set of data.</p>

NORTH DAKOTA  
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> <li>• Given a set of data exhibiting a linear trend, approximate an equation for the line of best fit (with or without technology) and use that model to make predictions</li> </ul>	<p>Given a set of data exhibiting a linear trend, students determine a reasonable equation for the line of best fit and use that model to make and defend meaningful predictions.</p>	<p>Given a set of data exhibiting a linear trend, students determine a reasonable equation for the line of best fit and use that model to make meaningful predictions.</p>	<p>Given a set of data exhibiting a linear trend, students determine a reasonable equation for the line of best fit, but make faulty or no predictions.</p>	<p>Given a set of data exhibiting a linear trend, students are unable to determine a reasonable equation for the line of best fit.</p>

## Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 11-12</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> <li>• No further expectations</li> </ul> <p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> <li>• No further expectations</li> </ul>				

## Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 11-12</p> <p><b>PATTERNS, RELATIONS, AND FUNCTIONS</b></p> <ul style="list-style-type: none"> <li>Perform advanced operations (i.e., composition and finding inverses) on algebraic functions</li> <li>Generate graphs of a variety of functions (i.e., linear, quadratic, polynomial, absolute value, and exponential), using technology when appropriate</li> </ul> <p><b>NUMERIC AND ALGEBRAIC REPRESENTATIONS</b></p> <ul style="list-style-type: none"> <li>Solve quadratic equations involving complex roots</li> <li>Use transformations (i.e., reflection, translation, dilation) to graph linear, quadratic, and absolute value functions</li> </ul>	<p>Students correctly perform advanced operations on algebraic functions.</p> <p>Students correctly graph a variety of functions.</p> <p>Students correctly solve quadratic equations involving complex roots.</p> <p>Students correctly graph linear, quadratic, and absolute value functions involving one transformation and provide additional analysis.</p>	<p>Students perform, with minor computational errors, advanced operations on algebraic functions.</p> <p>Students make minor errors when graphing a variety of functions.</p> <p>Students solve, with minor computational errors, quadratic equations involving complex roots.</p> <p>Students correctly graph linear, quadratic, and absolute value functions involving one transformation.</p>	<p>Students make procedural errors in attempting to perform advanced operations on algebraic functions.</p> <p>Students make substantial errors when graphing a variety of functions.</p> <p>Students solve, with minor procedural errors, quadratic equations involving complex roots.</p> <p>Students make a reasonable attempt to graph linear, quadratic, and absolute value functions involving one transformation.</p>	<p>Students are unable to perform advanced operations on algebraic functions.</p> <p>Students are unable to generate reasonable graphs of functions.</p> <p>Students are unable to solve quadratic equations involving complex roots.</p> <p>Students are unable to graph linear, quadratic, and absolute value functions involving one transformation.</p>

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>Given the graph of a transformed linear, quadratic, or absolute value function, write its equation</li> </ul>	<p>Given the graph of a linear, quadratic, or absolute value function involving a single transformation, students correctly write its equation and provide additional analysis.</p>	<p>Given the graph of a linear, quadratic, or absolute value function involving a single transformation, students correctly write its equation.</p>	<p>Given the graph of a linear, quadratic, or absolute value function involving a single transformation, students make a reasonable attempt to write its equation.</p>	<p>Given the graph of a linear, quadratic, or absolute value function involving a single transformation, students are unable to write its equation.</p>
<p><b>MATHEMATICAL MODELING</b></p> <ul style="list-style-type: none"> <li>Determine and write an equation for a function (i.e., linear, quadratic, polynomial, absolute value, and exponential) that models a mathematical relationship</li> </ul>	<p>Students demonstrate extensive understanding when selecting a function to model a mathematical relationship and writing an equation for the function.</p>	<p>Students demonstrate substantial understanding when selecting a function to model a mathematical relationship and writing an equation for the function.</p>	<p>Students demonstrate limited understanding when selecting a function to model a mathematical relationship and writing an equation for the function.</p>	<p>Students demonstrate a lack of understanding when selecting a function to model a mathematical relationship and writing an equation for the function.</p>
<p><b>RATES OF CHANGE</b></p> <ul style="list-style-type: none"> <li>No further expectations</li> </ul>				