

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

Content standard – A description of what students should know and be able to do within a particular content discipline or subject.

Subtopic - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

Benchmark expectation - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

Achievement standard A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

Advanced Proficient -Demonstrates exemplary understanding and exceeds expected level of performance

Proficient - Demonstrates understanding and meets expected level of performance.

Partially Proficient - Demonstrates an emerging or developing level of understanding and performance.

Novice – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 3</p> <p>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</p> <ul style="list-style-type: none"> • Count and order numbers up to 10,000 • Read and write numerals to 10,000 • Represent numbers up to 10,000 in standard, expanded, and word form • Identify the odd and even whole numbers from 0 to 10,000 • Identify place values from ten thousands through the hundredths place • Use the appropriate symbols to compare whole numbers from 0 to 10,000; i.e., $>$, $<$, $=$ • Use appropriate terms when communicating about computations; i.e., addend, sum, minuend, subtrahend, difference, factor, product, divisor, dividend, quotient, numerator, and denominator • Round numbers to tens, hundreds, and thousands 	<p>Students correctly count, order, read, write, round, compare and represent numbers up to 10,000 and apply to problem-solving situations.</p> <p>Students consistently use appropriate terms when communicating about computations.</p>	<p>Students correctly count, order, read, write, round, compare, and represent numbers up to 10,000.</p> <p>Students consistently use appropriate terms when communicating about computations with few errors.</p>	<p>Students count, order, read, write, round, compare, and represent numbers up to 10,000 with some errors.</p> <p>Students occasionally use appropriate terms when communicating about computations.</p>	<p>Students ineffectively count, order, read, write, round, compare, and represent numbers up to 10,000.</p> <p>Students rarely use appropriate terms when communicating about computations.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Represent fractions and mixed numbers using words, numerals, and physical models • Model, represent, and explain the concept of multiplication; i.e., repeated addition, rectangular arrays, and skip counting • Model, represent, and explain the concept of division; i.e., repeated subtraction, rectangular arrays, and equal sharing • Use a variety of methods and tools for problem solving; e.g., computing, including mental math, paper and pencil, calculator, manipulatives 	Students model, represent, and explain fractions, mixed numbers, multiplication, and division with a thorough and comprehensive understanding.	Students correctly model, represent, and adequately explain fractions, mixed numbers, multiplication, and division.	Students correctly model, represent, and adequately explain fractions, mixed numbers, multiplication, and division with limited understanding.	Students model, represent, and adequately explain fractions, mixed numbers, multiplication, and division with minimal understanding.
	<p style="text-align: center;">OPERATIONS AND THEIR PROPERTIES</p> <ul style="list-style-type: none"> • Model and use the commutative and associative properties of addition • Add and subtract whole numbers between 0 and 10,000 	Students accurately use a variety of methods and tools for problem solving. Students consistently add and subtract whole numbers between 0 and 10,000.	Students adequately use a variety of methods and tools for problem solving. Students consistently add and subtract whole numbers between 0 and 10,000 with few errors.	Students marginally use a variety of methods and tools for problem solving. Students occasionally add and subtract whole numbers between 0 and 10,000.

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Model and use the commutative and associative properties of multiplication • Apply the multiplication property of zero and one • Multiply two- and three-digit numbers by a single-digit number 	Students correctly model and use the commutative and associative properties of addition and multiplication with a thorough and comprehensive understanding.	Students correctly model and use the commutative and associative properties of addition and multiplication.	Students model and use the commutative and associative properties of addition and multiplication with limited understanding.	Students model and use the commutative and associative properties of addition and multiplication with minimal understanding.
	Students correctly apply and explain the multiplication properties of zero and one and accurately multiply and explain two and three digit numbers by a single-digit number with no errors.	Students correctly apply the multiplication properties of zero and one and accurately multiply two and three digit numbers by a single-digit number.	Students apply the multiplication properties of zero and one and multiply two and three digit numbers by a single-digit number with some errors.	Students incorrectly apply the multiplication properties of zero and one and inaccurately multiply two and three digit numbers by a single-digit number.

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Divide two- and three-digit numbers by a single-digit number without remainders • Demonstrate the inverse relationship between multiplication and division 	<p>Students demonstrate a deep understanding of the inverse relationship between multiplication and division and demonstrates mastery in the division of two and three digit numbers without remainders.</p>	<p>Students accurately demonstrate the inverse relationship between multiplication and division and correctly divide two and three digit numbers without remainders.</p>	<p>Students demonstrate the inverse relationship between multiplication and division and divide two and three digit numbers without remainders with some errors.</p>	<p>Students inaccurately demonstrate the inverse relationship between multiplication and division and incorrectly divide two and three digit numbers without remainders.</p>
	<p>Students consistently add and subtract simple fractions with like denominators.</p>	<p>Students consistently add and subtract simple fractions with like denominators with few errors.</p>	<p>Students occasionally add and subtract simple fractions with like denominators.</p>	<p>Students rarely add and subtract simple fractions with like denominators.</p>
<p>COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> • Recall multiplication and division facts 0-10 	<p>Students automatically recall multiplication and division facts from 0 to 10.</p>	<p>Students accurately recall multiplication and division facts from 0 to 10.</p>	<p>Students recall with some errors multiplication and division facts from 0 to 10.</p>	<p>Students rarely recall multiplication and division facts from 0 to 10.</p>

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Estimate whole number products and quotients • Use estimation to determine if solutions are reasonable 	<p>Students demonstrate mastery in estimating whole number products and quotients and in using estimation to determine if solutions are reasonable.</p>	<p>Students estimate whole number products and quotients and use estimation to determine if solutions are reasonable.</p>	<p>Students occasionally estimate whole number products and quotients and use estimation to determine if solutions are reasonable.</p>	<p>Students rarely estimate whole number products and quotients and use estimation to determine if solutions are reasonable.</p>

Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 3</p> <p>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</p> <ul style="list-style-type: none"> • Compare physical attributes of two-dimensional shapes; i.e., square, triangle, rectangle, and parallelogram • Describe the characteristics of a cylinder, pyramid, cube, sphere, and cone <ul style="list-style-type: none"> • Identify points, endpoints, lines, line segments, rays, and angles and use symbols to represent them • Identify right angles 	<p>Students accurately compare physical attributes of two-dimensional shapes and describe the characteristics of various three-dimensional shapes.</p> <p>Students correctly identify points, endpoints, lines, line segments, rays, and angles and use symbols to represent them.</p>	<p>Students adequately compare physical attributes of two-dimensional shapes and describe the characteristics of various three-dimensional shapes.</p> <p>Students consistently identify points, endpoints, lines, line segments, rays, and angles and use symbols to represent them with few errors.</p>	<p>Students omit important details when comparing physical attributes of two-dimensional shapes and when describing the characteristics of various three-dimensional shapes.</p> <p>Students identify points, endpoints, lines, line segments, rays, and angles and use symbols to represent them with some errors.</p>	<p>Students inaccurately compare physical attributes of two-dimensional shapes and inaccurately describe the characteristics of various three-dimensional shapes.</p> <p>Students incorrectly identify points, endpoints, lines, line segments, rays, and angles and use symbols to represent them.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> Use ordered pairs to identify the locations of points in a grid 	<p>Students accurately use ordered pairs to identify the locations of points in a grid.</p>	<p>Students consistently use ordered pairs to identify the locations of points in a grid with few errors.</p>	<p>Students use ordered pairs to identify the locations of points in a grid with some errors.</p>	<p>Students inaccurately use ordered pairs to identify the locations of points in a grid.</p>
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> Identify and create shapes that have lines of symmetry 	<p>Students correctly identify and create shapes that have lines of symmetry.</p>	<p>Students identify and create shapes that have lines of symmetry with few errors.</p>	<p>Students identify and create shapes that have lines of symmetry with some errors.</p>	<p>Students identify and create shapes that have lines of symmetry with many errors.</p>
<ul style="list-style-type: none"> Identify two-dimensional shapes that are congruent or similar Identify and describe slides, flips, and turns 	<p>Students accurately identify two-dimensional shapes that are congruent or similar, and identify and fully describe slides, flips, and turns.</p>	<p>Students consistently identify two-dimensional shapes that are congruent or similar, and identify and fully describe slides, flips, and turns.</p>	<p>Students occasionally identify two-dimensional shapes that are congruent or similar, and identify and fully describe slides, flips, and turns.</p>	<p>Students inaccurately identify two-dimensional shapes that are congruent or similar, and identify and fully describe slides, flips, and turns.</p>
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> No new expectations at this level 				

Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 3</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> • Identify different parts of a graph; i.e., label, scale, and data • Display and interpret graphs with symbols or pictures that represent more than one object or event • Solve problems based on data displayed on a graph <ul style="list-style-type: none"> • Recognize the elements in the union and intersection of sets representing Venn diagrams 	<p>Students correctly identify parts of a graph, accurately display and interpret graphs with symbols or pictures that represent more than one object or event, and correctly solve problems based on data displayed on a graph.</p> <p>Students correctly recognize the elements in the union and intersection of sets representing Venn diagrams.</p>	<p>Students identify parts of a graph, display and interpret graphs with symbols or pictures that represent more than one object or event, and solve problems based on data displayed on a graph with few errors.</p> <p>Students recognize the elements in the union and intersection of sets representing Venn diagrams</p>	<p>Students identify parts of a graph, display and interpret graphs with symbols or pictures that represent more than one object or event, and solve problems based on data displayed on a graph with some errors.</p> <p>Students recognize the elements in the union and intersection of sets representing Venn diagrams with some errors.</p>	<p>Students incorrectly identify parts of a graph, inaccurately display and interpret graphs with symbols or pictures that represent more than one object or event, and incorrectly solve problems based on data displayed on a graph.</p> <p>Students incorrectly recognize the elements in the union and intersection of sets representing Venn diagrams.</p>

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>PROBABILITY</p> <ul style="list-style-type: none"> Use a simple probability experiment to collect data, display the data in a graph, and interpret the likelihood of the outcome 	<p>Students competently use a simple probability experiment to collect data, display the data in a graph, and interpret the likelihood of the outcome.</p>	<p>Students adequately use a simple probability experiment to collect data, display the data in a graph, and interpret the likelihood of the outcome.</p>	<p>Students use a simple probability experiment to collect data, display the data in a graph, and interpret the likelihood of the outcome with inaccuracies.</p>	<p>Students incorrectly use a simple probability experiment to collect data, display the data in a graph, and interpret the likelihood of the outcome.</p>
<p>STATISTICAL METHODS</p> <ul style="list-style-type: none"> Identify the mode in a data set 	<p>Students automatically identify the mode in a data set.</p>	<p>Students quickly identify the mode in a data set.</p>	<p>Students haltingly identify the mode in a data set.</p>	<p>Students slowly identify the mode in a data set.</p>
<p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> Determine which outcomes are most likely to occur in certain situations; e.g., spinning red is most likely to occur when the spinner is divided among red, blue, green, red 	<p>Students accurately explain which outcomes are most likely to occur in certain situations.</p>	<p>Students adequately explain which outcomes are most likely to occur in certain situations.</p>	<p>Students generally explain which outcomes are most likely to occur in certain situations.</p>	<p>Students inaccurately explain which outcomes are most likely to occur in certain situations.</p>

Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 3</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> • Tell time to the nearest minute using digital and analog clocks • Determine elapsed time by the hour • Count coins and bills • Read and measure temperature with a thermometer using Fahrenheit and Celsius scales • Estimate and measure to the nearest half inch or centimeter • State specific relationships between units within the same measuring system; e.g., hours in a day, inches in a foot, cups in a pint 	<p>Students correctly tell time to the nearest minute and accurately determine elapsed time by the hour.</p> <p>Students accurately count coins and bills and correctly read and measure temperature.</p> <p>Students reasonably estimate and measure to the nearest half inch or centimeter and correctly state specific relationships between units within measuring systems.</p>	<p>Students tell time to the nearest minute and determine elapsed time by the hour with few errors.</p> <p>Students count coins and bills and read and measure temperature with few errors.</p> <p>Students adequately estimate and measure to the nearest half inch or centimeter and state specific relationships between units within measuring systems.</p>	<p>Students tell time to the nearest minute and determine elapsed time by the hour with some errors.</p> <p>Students count coins and bills and read and measure temperature with some errors.</p> <p>Students occasionally estimate and measure to the nearest half inch or centimeter and state specific relationships between units within measuring systems.</p>	<p>Students incorrectly tell time to the nearest minute and inaccurately determine elapsed time by the hour.</p> <p>Students inaccurately count coins and bills and incorrectly read and measure temperature.</p> <p>Students rarely estimate and measure to the nearest half inch or centimeter and state specific relationships between units within measuring systems.</p>

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Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> Estimate and measure perimeter, area, and volume using links, tiles, grid paper, geoboards, and dot paper 	<p>Students reasonably estimate and correctly measure perimeter, area, and volume.</p>	<p>Students adequately estimate and measure perimeter, area, and volume.</p>	<p>Students occasionally estimate and measure perimeter, area, and volume.</p>	<p>Students rarely estimate and measure perimeter, area, and volume.</p>
<p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> Select a variety of tools for measuring length, weight, and capacity 	<p>Students automatically select appropriate tools for measuring length, weight, and capacity.</p>	<p>Students quickly select appropriate tools for measuring length, weight, and capacity.</p>	<p>Students haltingly select appropriate tools for measuring length, weight, and capacity.</p>	<p>Students slowly select appropriate tools for measuring length, weight, and capacity.</p>

Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 3</p> <p>PATTERNS, RELATIONS, AND FUNCTIONS</p> <ul style="list-style-type: none"> • Use patterns to solve problems • Create patterns using multiplication • Determine the missing elements of a pattern of multiples <p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> • Solve addition, subtraction, multiplication, and division equations with unknown numbers; e.g., $8 \times \square = 56$ <p>MATHEMATICAL MODELING</p> <ul style="list-style-type: none"> • Use symbols to write number sentences; i.e., $+$, $-$, $>$, $<$, $=$, \times, and \div 	<p>Students competently use patterns to solve problems.</p> <p>Students correctly create patterns using multiplication and determine the missing elements of a pattern of multiples.</p> <p>Students correctly solve addition, subtraction, multiplication, and division equations with unknown numbers.</p> <p>Students correctly use symbols to write number sentences.</p>	<p>Students adequately use patterns to solve problems.</p> <p>Students create patterns using multiplication and determine the missing elements of a pattern of multiples with few errors.</p> <p>Students solve addition, subtraction, multiplication, and division equations with unknown numbers with few errors.</p> <p>Students use symbols to write number sentences with few errors.</p>	<p>Students use patterns to solve problems with some errors.</p> <p>Students create patterns using multiplication and determine the missing elements of a pattern of multiples with some errors.</p> <p>Students solve addition, subtraction, multiplication, and division equations with unknown numbers with some errors.</p> <p>Students use symbols to write number sentences with some errors.</p>	<p>Students use patterns to solve problems with many errors.</p> <p>Students create patterns using multiplication and determine the missing elements of a pattern of multiples with many errors.</p> <p>Students solve addition, subtraction, multiplication, and division equations with unknown numbers with many errors.</p> <p>Students use symbols to write number sentences with many errors.</p>

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Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
RATES OF CHANGE <ul style="list-style-type: none"> • No new expectations at this level 				