

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

**Content standard** – A description of what students should know and be able to do within a particular content discipline or subject.

**Subtopic** - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

**Benchmark expectation** - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

**Achievement standard** A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

**Advanced Proficient** -Demonstrates exemplary understanding and exceeds expected level of performance

**Proficient** - Demonstrates understanding and meets expected level of performance.

**Partially Proficient** - Demonstrates an emerging or developing level of understanding and performance.

**Novice** – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

## Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 4</p> <p>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</p> <ul style="list-style-type: none"> <li>• Identify Roman numerals; i.e., I, V, X, L, C, D, M</li> <li>• Identify place value from hundred thousands through the thousandths place</li> <li>• Order and compare using symbols (<math>&gt;</math>, <math>&lt;</math>, <math>=</math>) whole numbers (0 to 100,000) and decimals to hundredths</li> <li>• Round whole numbers to the nearest tens, hundreds, thousands, ten thousands, and hundred thousands</li> <li>• Read and write numerals to 100,000</li> <li>• Represent numbers up to hundred thousands in standard and expanded forms</li> <li>• Write tenths and hundredths as decimals and fractions</li> <li>• Compare equivalent decimals and fractions; e.g., <math>.5=1/2</math></li> </ul>	<p>Students correctly count, order, read, write, round, compare, and represent numbers up to 100,000 and apply to problem-solving situations.</p> <p>Students accurately write tenths and hundredths as decimals and fractions and correctly compare equivalent decimals and fractions.</p>	<p>Students correctly count, order, read, write, round, compare, and represent numbers up to 100,000.</p> <p>Students write tenths and hundredths as decimals and fractions and compare equivalent decimals and fractions with few errors.</p>	<p>Students count, order, read, write, round, compare, and represent numbers up to 100,000 with some errors.</p> <p>Students write tenths and hundredths as decimals and fractions and compare equivalent decimals and fractions with some errors.</p>	<p>Students incorrectly count, order, read, write, round, compare, and represent numbers up to 100,000.</p> <p>Students write tenths and hundredths as decimals and fractions and compare equivalent decimals and fractions with many errors.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Use appropriate terms when communicating about computations; i.e., addend, sum, minuend, subtrahend, difference, factor, product, divisor, dividend, and quotient</li>   <li>• Explain the meaning of remainders in real-world situations</li>   <li>• Determine what information is relevant for solving a problem</li>   <li>• Use a variety of strategies to solve problems; e.g., guess and check, work backwards, draw pictures, use objects</li> </ul>	Students use appropriate terms consistently when communicating about computations.	Students use appropriate terms consistently, and with few errors, when communicating about computations.	Students occasionally use appropriate terms when communicating about computations.	Students rarely use appropriate terms when communicating about computations.
	Students understand and explain the meaning of remainders in real-world situations.	Students correctly explain the meaning of remainders in real-world situations.	Students generally explain the meaning of remainders in real-world situations.	Students incorrectly explain the meaning of remainders in real-world situations.
	Students consistently determine what information is relevant for solving a problem and automatically use a variety of strategies to solve problems.	Students consistently, and with few errors, determine what information is relevant for solving a problem and correctly use a variety of strategies to solve problems.	Students occasionally, and with some errors, determine what information is relevant for solving a problem and use a variety of strategies to solve problems.	Students rarely, and with many errors, determine what information is relevant for solving a problem and incorrectly use a variety of strategies to solve problems

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Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>OPERATIONS AND THEIR PROPERTIES</p> <ul style="list-style-type: none"> <li>• Add and subtract whole numbers between 0 and 100,000</li>   <li>• Multiply multi-digit numbers by two-digit numbers</li> <li>• Divide multi-digit numbers by a single-digit number</li>   <li>• Add and subtract fractions and mixed numbers with like denominators</li> <li>• Add and subtract decimals</li> </ul>	<p>Students consistently add and subtract whole numbers between 0 and 100,000.</p> <p>Students accurately multiply multi-digit numbers by two-digit numbers and divide multi-digit numbers by single-digit numbers.</p> <p>Students accurately add and subtract fractions and mixed numbers with like denominator and add and subtract decimals.</p>	<p>Students consistently add and subtract whole numbers between 0 and 100,000 with few errors.</p> <p>Students consistently multiply multi-digit numbers by two-digit numbers and divide multi-digit numbers by single-digit numbers with few errors.</p> <p>Students consistently add and subtract fractions and mixed numbers with like denominator and add and subtract decimals with few errors.</p>	<p>Students occasionally add and subtract whole numbers between 0 and 100,000.</p> <p>Students occasionally multiply multi-digit numbers by two-digit numbers and divide multi-digit numbers by single-digit numbers with some errors.</p> <p>Students occasionally add and subtract fractions and mixed numbers with like denominator and add and subtract decimals with some errors.</p>	<p>Students rarely add and subtract whole numbers between 0 and 100,000.</p> <p>Students multiply multi-digit numbers by two-digit numbers and divide multi-digit numbers by single-digit numbers with many errors.</p> <p>Students add and subtract fractions and mixed numbers with like denominator and add and subtract decimals with many errors.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
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<ul style="list-style-type: none"> <li>Use the distributive property to simplify and perform computations</li> </ul>	<p>Students correctly and consistently use the distributive property to simplify and perform computations with a thorough and comprehensive understanding.</p>	<p>Students correctly and consistently use the distributive property to simplify and perform computations.</p>	<p>Students occasionally use the distributive property to simplify and perform computations.</p>	<p>Students incorrectly and inconsistently use the distributive property to simplify and perform computations.</p>
<p>COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> <li>Determine when a rounded solution is appropriate</li> <li>Estimate computations of whole numbers, fractions, and decimals</li> </ul>	<p>Students reasonably estimate computations of whole numbers, fractions and decimals and consistently and correctly determine when a rounded solution is appropriate.</p>	<p>Students reasonably estimate computations of whole numbers, fractions and decimals with few errors and often determine when a rounded solution is appropriate.</p>	<p>Students reasonably estimate computations of whole numbers, fractions and decimals with some errors and occasionally determine when a rounded solution is appropriate.</p>	<p>Students reasonably estimate computations of whole numbers, fractions and decimals with many errors and rarely determine when a rounded solution is appropriate.</p>

## Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 4</p> <p><b>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</b></p> <ul style="list-style-type: none"> <li>Analyze the attributes of two- and three-dimensional shapes (i.e., circle, squares, trapezoid, rhombus) and use vocabulary to describe the attributes</li> <li>Identify, describe, and model (e.g., using straws or other materials) parallel, perpendicular, and intersecting lines and line segments</li> </ul> <p><b>COORDINATE GEOMETRY</b></p> <ul style="list-style-type: none"> <li>Create objects using ordered pairs in a grid</li> </ul>	<p>Students describe in accurate detail the attributes of two- and three-dimensional shapes and accurately use the vocabulary.</p> <p>Students correctly identify, describe, and model parallel, perpendicular, and intersecting lines and line segments.</p> <p>Students accurately create objects using ordered pairs in a grid.</p>	<p>Students describe in adequate detail the attributes of two- and three-dimensional shapes and adequately use the vocabulary.</p> <p>Students adequately identify, describe, and model parallel, perpendicular, and intersecting lines and line segments.</p> <p>Students consistently create objects using ordered pairs in a grid with few errors.</p>	<p>Students describe with limited detail the attributes of two- and three-dimensional shapes and have a limited use the vocabulary.</p> <p>Students occasionally identify, describe, and model parallel, perpendicular, and intersecting lines and line segments.</p> <p>Students create objects using ordered pairs in a grid with some errors.</p>	<p>Students rarely describe the attributes of two- and three-dimensional shapes and inadequately use the vocabulary.</p> <p>Students rarely identify, describe, and model parallel, perpendicular, and intersecting lines and line segments.</p> <p>Students inaccurately create objects using ordered pairs in a grid.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> <li>• Recognize the changes in position and orientation of two-dimensional figures after transformations; i.e., flips (reflections), turns (rotations), and slides (translations)</li> <li>• Use motion geometry to show that shapes are congruent or similar</li> </ul>	<p>Students accurately recognize the changes in position and orientation of two-dimensional figures after transformations and use motion geometry to show that shapes are congruent or similar.</p>	<p>Students consistently recognize the changes in position and orientation of two-dimensional figures after transformations and use motion geometry to show that shapes are congruent or similar with few errors.</p>	<p>Students occasionally recognize the changes in position and orientation of two-dimensional figures after transformations and use motion geometry to show that shapes are congruent or similar with some errors.</p>	<p>Students rarely recognize the changes in position and orientation of two-dimensional figures after transformations and correctly use motion geometry to show that shapes are congruent or similar with many errors.</p>
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> <li>• No new expectations at this level</li> </ul>				

### Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 4</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> <li>• Determine a sample group to survey</li> <li>• Collect and record data</li> <li>• Organize and display data in line graphs and circle graphs</li> <li>• Read, interpret, and generate questions from data displayed in graphs; i.e., line graphs and circle graphs</li> <li>• Use computers and spread sheets to organize and display data</li> <li>• Use number lines and coordinate graphs to represent data</li> </ul>	<p>Students demonstrate mastery in collecting, recording, organizing, and displaying data in line graphs and circle graphs.</p> <p>Students accurately read, interpret, and generate questions that show a comprehensive understanding of data displayed in graphs.</p> <p>Students correctly use number lines and coordinate graphs to represent data and correctly use computers and spread sheets to organize and display data.</p>	<p>Students demonstrate competent performance in collecting, recording, organizing, and displaying data in line graphs and circle graphs.</p> <p>Students adequately read, interpret, and generate questions that show a substantial understanding of data displayed in graphs.</p> <p>Students use number lines and coordinate graphs to represent data and use computers and spread sheets to organize and display data with few errors.</p>	<p>Students collect, record, organize, and display data in line graphs and circle graphs without demonstrating complete understanding.</p> <p>Students occasionally read, interpret, and generate questions that show a limited understanding of data displayed in graphs.</p> <p>Students use number lines and coordinate graphs to represent data and use computers and spread sheets to organize and display data with some errors.</p>	<p>Students demonstrate incompetent understanding in collecting, recording, organizing, and displaying data in line graphs and circle graphs.</p> <p>Students read, interpret, and generate questions that show a minimal understanding of data displayed in graphs.</p> <p>Students incorrectly use number lines and coordinate graphs to represent data and incorrectly use computers and spread sheets to organize and display data.</p>

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Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p><b>PROBABILITY</b></p> <ul style="list-style-type: none"> <li>Construct simple probability experiments</li> </ul>	<p>Students accurately construct simple probability experiments.</p>	<p>Students construct simple probability experiments with few errors.</p>	<p>Students construct simple probability experiments with some errors.</p>	<p>Students inaccurately construct simple probability experiments.</p>
<p><b>STATISTICAL METHODS</b></p> <ul style="list-style-type: none"> <li>Determine or calculate the mode and mean/average for a data set</li> </ul>	<p>Students competently determine or calculate the mode and mean for a data set.</p>	<p>Students consistently with few errors determine or calculate the mode and mean for a data set.</p>	<p>Students occasionally determine or calculate the mode and mean for a data set.</p>	<p>Students inconsistently determine or calculate the mode and mean for a data set.</p>
<p><b>PREDICTIONS, DATA ANALYSIS AND INFERENCES</b></p> <ul style="list-style-type: none"> <li>Make predictions and draw conclusions from simple probability experiments</li> </ul>	<p>Students accurately make predictions and draw appropriate conclusions from simple probability experiments.</p>	<p>Students make predictions and draw appropriate conclusions from simple probability experiments with few errors.</p>	<p>Students occasionally make predictions and draw appropriate conclusions from simple probability experiments with some errors.</p>	<p>Students inaccurately make predictions and draw appropriate conclusions from simple probability experiments.</p>

## Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 4</p> <p><b>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</b></p> <ul style="list-style-type: none"> <li>• State specific relationships between units within the same measuring system; e.g., feet to yards, minutes to hours, milliliters to liters</li>   <li>• Estimate and measure length to the nearest quarter inch</li>   <li>• Analyze relationships between perimeter and area</li>   <li>• Make change up to \$20</li>   <li>• Apply the concept of elapsed time; i.e., schedules, and calendars</li> </ul>	<p>Students demonstrate mastery in identifying appropriate connections between units within the same measuring systems.</p> <p>Students reasonably estimate and measure length to the nearest quarter inch and analyze relationships between perimeter and area.</p> <p>Students correctly make change up to \$20.</p> <p>Students competently apply the concept of elapsed time to schedules and calendars.</p>	<p>Students adequately identify appropriate connections between units within the same measuring systems.</p> <p>Students adequately estimate and measure length to the nearest quarter inch and analyze relationships between perimeter and area.</p> <p>Students consistently make change up to \$20 with few errors.</p> <p>Students apply the concept of elapsed time to schedules and calendars with few errors.</p>	<p>Students occasionally identify appropriate connections between units within the same measuring systems.</p> <p>Students occasionally estimate and measure length to the nearest quarter inch and analyze relationships between perimeter and area.</p> <p>Students make change up to \$20 with some errors.</p> <p>Students apply the concept of elapsed time to schedules and calendars with some errors.</p>	<p>Students rarely identify appropriate connections between units within the same measuring systems.</p> <p>Students rarely estimate and measure length to the nearest quarter inch and analyze relationships between perimeter and area.</p> <p>Students make change up to \$20 with many errors.</p> <p>Students apply the concept of elapsed time to schedules and calendars with many errors.</p>

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Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS <ul style="list-style-type: none"> <li>• Select appropriate units for measuring perimeter, area, and volume</li> </ul>	Students automatically select appropriate units for measuring perimeter, area, and volume with few errors.	Students quickly select appropriate units for measuring perimeter, area, and volume with few errors.	Students haltingly select appropriate units for measuring perimeter, area, and volume with few errors.	Students slowly select appropriate units for measuring perimeter, area, and volume with few errors.

## Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 4</p> <p><b>PATTERNS, RELATIONS, AND FUNCTIONS</b></p> <ul style="list-style-type: none"> <li>Determine the missing elements of complex repeating patterns</li> </ul> <p><b>NUMERIC AND ALGEBRAIC REPRESENTATIONS</b></p> <ul style="list-style-type: none"> <li>Explain that variables represent unknowns</li> </ul> <p><b>MATHEMATICAL MODELING</b></p> <ul style="list-style-type: none"> <li>Solve problems with variables</li> <li>Use parentheses in solving simple equations</li> </ul>	<p>Students correctly determine the missing elements of complex repeating patterns.</p> <p>Students competently understand and explain that variables represent unknowns.</p> <p>Students correctly solve problems with variables. Students correctly use parentheses in solving simple equations.</p>	<p>Students determine the missing elements of complex repeating patterns with few errors.</p> <p>Students adequately explain that variables represent unknowns.</p> <p>Students solve problems with variables with few errors. Students use parentheses in solving simple equations with few errors.</p>	<p>Students determine the missing elements of complex repeating patterns with some errors.</p> <p>Students have a limited understanding that variables represent unknowns.</p> <p>Students solve problems with variables with some errors. Students use parentheses in solving simple equations with some errors.</p>	<p>Students determine the missing elements of complex repeating patterns with many errors.</p> <p>Students have a minimal understanding that variables represent unknowns.</p> <p>Students solve problems with variables with many errors. Students use parentheses in solving simple equations with many errors.</p>

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Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
RATES OF CHANGE <ul style="list-style-type: none"> <li>• No new expectations at this level</li> </ul>				