

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

Content standard – A description of what students should know and be able to do within a particular content discipline or subject.

Subtopic - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

Benchmark expectation - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

Achievement standard A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

Advanced Proficient -Demonstrates exemplary understanding and exceeds expected level of performance

Proficient - Demonstrates understanding and meets expected level of performance.

Partially Proficient - Demonstrates an emerging or developing level of understanding and performance.

Novice – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Round, order, and compare using symbols fractions with like and unlike denominators • Round, order, and compare using symbols decimals to the tenths, hundredths, and thousandths place 	Students correctly round, order, and compare fractions with like and unlike denominators and round, order, and compare decimals to the tenths, hundredths, and thousandths place.	Students round, order, and compare fractions with like and unlike denominators and round, order, and compare decimals to the tenths, hundredths, and thousandths place with few errors.	Students round, order, and compare fractions with like and unlike denominators and round, order, and compare decimals to the tenths, hundredths, and thousandths place with some errors.	Students round, order, and compare fractions with like and unlike denominators and round, order, and compare decimals to the tenths, hundredths, and thousandths place with many errors.
	<ul style="list-style-type: none"> • Explain and demonstrate the concept of a percent • Compare equivalent fractions, decimals, and percents • Represent ratios and percents as parts of a whole using models and pictures 	Students accurately explain and demonstrate the concept of a percent, correctly compare equivalent fractions, decimals, and percents, and correctly represent ratios and percents as parts.	Students adequately demonstrate the concept of a percent, compare equivalent fractions, decimals, and percents, and represent ratios and percents as parts with few errors.	Students occasionally demonstrate the concept of a percent, compare equivalent fractions, decimals, and percents, and represent ratios and percents as parts without complete understanding.

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> Explain and demonstrate the relationship between exponential notation and repeated multiplication; e.g., $3^2 = 3 \times 3$ 	<p>Students effectively explain and demonstrate a substantial understanding of the relationship between exponential notations and repeated multiplication.</p>	<p>Students demonstrate a substantial understanding of the relationship between exponential notations and repeated multiplication.</p>	<p>Students occasionally demonstrate an understanding of the relationship between exponential notations and repeated multiplication.</p>	<p>Students ineffectively demonstrate an understanding of the relationship between exponential notations and repeated multiplication.</p>
OPERATIONS AND THEIR PROPERTIES				
<ul style="list-style-type: none"> Add and subtract whole numbers between 0 and 1,000,000 Use commutative, associative, and identity properties Use divisibility rules for 2, 5, and 10 	<p>Students consistently add and subtract whole numbers between 0 and 1,000,000.</p> <p>Students correctly use commutative, associative, and identify properties and accurately use divisibility rules for 2, 5, and 10.</p>	<p>Students consistently add and subtract whole numbers between 0 and 1,000,000 with few errors.</p> <p>Students use commutative, associative, and identify properties and use divisibility rules for 2, 5, and 10 with few errors.</p>	<p>Students occasionally add and subtract whole numbers between 0 and 1,000,000.</p> <p>Students use commutative, associative, and identify properties and use divisibility rules for 2, 5, and 10 with some errors.</p>	<p>Students rarely add and subtract whole numbers between 0 and 1,000,000.</p> <p>Students use commutative, associative, and identify properties and use divisibility rules for 2, 5, and 10 with many errors.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Determine the prime factors for a number using a factor tree • Determine least common multiple • Determine greatest common factor 	Students accurately determine the prime factors for a number using a factor tree and accurately determine the least common multiple and greatest common factor.	Students consistently determine the prime factors for a number using factor tree and consistently determine the least common multiple and greatest common factor with few errors.	Students determine the prime factors for a number using factor tree and determine the least common multiple and greatest common factor with some errors.	Students inconsistently determine the prime factors for a number using factor tree and inconsistently determine the least common multiple and greatest common factor.
	Students correctly use the order of operations to simplify numeric expressions.	Students use the order of operations to simplify numeric expressions with few errors.	Students use the order of operations to simplify numeric expressions with some errors.	Students use the order of operations to simplify numeric expressions with many errors.
	Students accurately multiply multi-digit numbers by three-digit numbers and divide multi-digit numbers by two-digit numbers with or without remainders.	Students consistently multiply multi-digit numbers by three-digit numbers and divide multi-digit numbers by two-digit numbers with or without remainders with few errors.	Students occasionally multiply multi-digit numbers by three-digit numbers and divide multi-digit numbers by two-digit numbers with or without remainders with some errors.	Students multiply multi-digit numbers by three-digit numbers and divide multi-digit numbers by two-digit numbers with or without remainders with many errors.
<ul style="list-style-type: none"> • Use order of operations to simplify numeric expressions • Multiply multi-digit numbers by three-digit numbers • Divide multi-digit numbers by two-digit numbers with or without remainders 				

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Add and subtract improper fractions and mixed numbers with unlike denominators • Add and subtract multi-digit decimals • Multiply and divide multi-digit decimals <p>COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> • No new content at this level 	<p>Students accurately add and subtract improper fractions and mixed numbers with unlike denominators.</p> <p>Students accurately add, subtract, multiply, and divide multi-digit decimals.</p>	<p>Students consistently add and subtract improper fractions and mixed numbers with unlike denominators with few errors.</p> <p>Students consistently add, subtract, multiply, and divide multi-digit decimals with few errors.</p>	<p>Students occasionally add and subtract improper fractions and mixed numbers with unlike denominators with some errors.</p> <p>Students occasionally add, subtract, multiply, and divide multi-digit decimals with some errors.</p>	<p>Students add and subtract improper fractions and mixed numbers with unlike denominators with many errors.</p> <p>Students add, subtract, multiply, and divide multi-digit decimals with many errors.</p>

Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 5</p> <p>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</p> <ul style="list-style-type: none"> • Describe properties and attributes of two- and three-dimensional figures; i.e., obtuse angle, acute angle, radius, chord, diagonal, equilateral triangle, isosceles triangle, parallel lines, perpendicular lines • Draw circles using a compass, and identify the components; i.e., radius, chord, diameter, center, and circumference • Identify the attributes of an angle and draw angles using protractors • Determine the degrees of the interior angles of triangles and quadrilaterals • Determine the characteristics of, and the relationships among, points, lines, line segments, rays, and planes <p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> • Create objects using ordered pairs in quadrant 1 	<p>Students describe properties in comprehensive detail properties and attributes of two- and three-dimensional figures.</p> <p>Students demonstrate mastery in using a compass and protractor.</p> <p>Students accurately determine the degrees of the interior angles of triangles and quadrilaterals.</p> <p>Students accurately create objects using ordered pairs in quadrant 1.</p>	<p>Students describe in adequate detail properties and attributes of two- and three-dimensional figures.</p> <p>Students demonstrate competent performance in using a compass and protractor.</p> <p>Students determine the degrees of the interior angles of triangles and quadrilaterals with few errors.</p> <p>Students create objects using ordered pairs in quadrant 1 with few errors.</p>	<p>Students describe with limited detail properties and attributes of two- and three-dimensional figures.</p> <p>Students demonstrate limited understanding in using a compass and protractor.</p> <p>Students determine the degrees of the interior angles of triangles and quadrilaterals with many errors.</p> <p>Students create objects using ordered pairs in quadrant 1 with some errors.</p>	<p>Students describe with minimal detail properties and attributes of two- and three-dimensional figures.</p> <p>Students incorrectly use a compass and protractor.</p> <p>Students inaccurately determine the degrees of the interior angles of triangles and quadrilaterals.</p> <p>Students inaccurately create objects using ordered pairs in quadrant 1.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> • Describe properties of congruent figures and use them to solve problems 	<p>Students accurately describe in detail properties of congruent figures and use them correctly to solve problems.</p>	<p>Students adequately describe in detail properties of congruent figures and use them to solve problems with few errors.</p>	<p>Students describe with limited detail properties of congruent figures and use them to solve problems with some errors.</p>	<p>Students inaccurately describe in detail properties of congruent figures and use them incorrectly to solve problems.</p>
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> • No new expectations at this level 				

Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 5</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> Read and interpret bar, line, and circle graphs, pictographs, and frequency tables <p>PROBABILITY</p> <ul style="list-style-type: none"> Determine the probability of a simple event and express it as a ratio State possible outcomes for a given situation <ul style="list-style-type: none"> Determine possible arrangements of four or fewer items 	<p>Students demonstrate mastery in reading and interpreting bar, line, and circle graphs, pictographs, and frequency tables.</p> <p>Students competently determine the probability of a simple event and express it as a ratio and correctly state possible outcomes for a given situation.</p> <p>Students automatically determine all possible arrangements of four or fewer items.</p>	<p>Students accurately read and interpret bar, line, and circle graphs, pictographs, and frequency tables.</p> <p>Students adequately determine the probability of a simple event and express it as a ratio and state possible outcomes for a given situation.</p> <p>Students adequately determine possible arrangements of four or fewer items.</p>	<p>Students read and interpret bar, line, and circle graphs, pictographs, and frequency tables with some errors.</p> <p>Students occasionally determine the probability of a simple event and express it as a ratio and state possible outcomes for a given situation with some errors.</p> <p>Students determine possible arrangements of four or fewer items with some errors.</p>	<p>Students inaccurately read and interpret bar, line, and circle graphs, pictographs, and frequency tables.</p> <p>Students rarely determine the probability of a simple event and express it as a ratio and state possible outcomes for a given situation.</p> <p>Students determine possible arrangements of four or fewer items with many errors.</p>

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>STATISTICAL METHODS</p> <ul style="list-style-type: none"> Determine or calculate the mode, mean, and range of a set of data <p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> Make predictions and draw conclusions based on data collected from a sample group 	<p>Students accurately determine or calculate the mode, mean, and range of a set of data.</p> <p>Students accurately make predictions and draw conclusions based on data collected from a sample group.</p>	<p>Students determine or calculate the mode, mean, and range of a set of data with few errors.</p> <p>Students make predictions and draw conclusions based on data collected from a sample group with few errors.</p>	<p>Students determine or calculate the mode, mean, and range of a set of data with some errors.</p> <p>Students make predictions and draw conclusions based on data collected from a sample group with some errors.</p>	<p>Students inaccurately determine or calculate the mode, mean, and range of a set of data.</p> <p>Students inaccurately make predictions and draw conclusions based on data collected from a sample group.</p>

Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 5</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> • Estimate and measure length to the nearest eighth inch • Measure and apply elapsed time; i.e., time zones, schedules, and calendars • Measure angles using protractors • Estimate angle measures using the benchmark angles 45°, 90°, 180°, 270°, and 360° <p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> • Select and use appropriate units when measuring length, area, and volume • Use formulas to calculate the perimeter and area of squares and rectangles 	<p>Students reasonably estimate and measure length to the nearest eighth inch and correctly measure and apply elapsed time.</p> <p>Students competently measure angles using protractors and estimate angle measures using the bench mark angles.</p> <p>Students automatically select and use appropriate units when measuring length, area, and volume with few errors.</p>	<p>Students reasonably estimate and measure length to the nearest eighth inch and measure and apply elapsed time with few errors.</p> <p>Students consistently measure angles using protractors and estimate angle measures using the bench mark angles with few errors.</p> <p>Students quickly select and use appropriate units when measuring length, area, and volume with few errors.</p>	<p>Students estimate and measure length to the nearest eighth inch, and measure and apply elapsed time with some errors.</p> <p>Students measure angles using protractors and estimate angle measures using the bench mark angles with some errors.</p> <p>Students haltingly select and use appropriate units when measuring length, area, and volume with few errors.</p>	<p>Students estimate and measure length to the nearest eighth inch, and measure and apply elapsed time with many errors.</p> <p>Students measure angles using protractors and estimate angle measures using the bench mark angles with many errors.</p> <p>Students slowly select and use appropriate units when measuring length, area, and volume with few errors.</p>

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Use formulas to calculate the volume of rectangular prisms 	<p>Students correctly use formulas to calculate the perimeter and area of squares and rectangles and the volume of rectangular prisms.</p>	<p>Students use formulas to calculate the perimeter and area of squares and rectangles and the volume of rectangular prisms with few errors.</p>	<p>Students use formulas to calculate the perimeter and area of squares and rectangles and the volume of rectangular prisms with some errors.</p>	<p>Students use formulas to calculate the perimeter and area of squares and rectangles and the volume of rectangular prisms with many errors.</p>

Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 5</p> <p>PATTERNS, RELATIONS, AND FUNCTIONS</p> <ul style="list-style-type: none"> • Analyze patterns represented by tables and graphs • Identify a rule for a pattern involving addition, subtraction, or multiplication • Identify the rule for a pattern and then use the rule to solve a problem <p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> • Identify a variable in an expression <p>MATHEMATICAL MODELING</p> <ul style="list-style-type: none"> • Use equations to solve problems; e.g., $28/x=7$ 	<p>Students analyze patterns represented by tables and graphs showing comprehensive understanding.</p> <p>Students correctly identify a rule for a pattern involving addition, subtraction, or multiplication and use the rule to solve a problem.</p> <p>Students accurately identify a variable in an expression.</p> <p>Students correctly use equations to solve problems.</p>	<p>Students analyze patterns represented by tables and graphs showing substantial understanding.</p> <p>Students identify a rule for a pattern involving addition, subtraction, or multiplication and use the rule to solve a problem with few errors.</p> <p>Students identify a variable in an expression with few errors.</p> <p>Students use equations to solve problems with few errors.</p>	<p>Students analyze patterns represented by tables and graphs showing limited understanding.</p> <p>Students identify a rule for a pattern involving addition, subtraction, or multiplication and use the rule to solve a problem with some errors.</p> <p>Students identify a variable in an expression with some errors</p> <p>Students use equations to solve problems with some errors.</p>	<p>Students analyze patterns represented by tables and graphs showing minimal understanding.</p> <p>Students identify a rule for a pattern involving addition, subtraction, or multiplication and use the rule to solve a problem with many errors.</p> <p>Students identify a variable in an expression with many errors.</p> <p>Students use equations to solve problems with many errors.</p>

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
RATES OF CHANGE <ul style="list-style-type: none"> • No new expectations at this level 				