

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

Content standard – A description of what students should know and be able to do within a particular content discipline or subject.

Subtopic - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

Benchmark expectation - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

Achievement standard A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

Advanced Proficient -Demonstrates exemplary understanding and exceeds expected level of performance

Proficient - Demonstrates understanding and meets expected level of performance.

Partially Proficient - Demonstrates an emerging or developing level of understanding and performance.

Novice – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Compare and order fractions, decimals, and mixed numbers • Generate a list of factors, prime factors, and multiples • Use rules to determine divisibility by 2, 3, 5, 6, 9, and 10. • Compare and order integers <p>OPERATIONS AND THEIR PROPERTIES</p> <ul style="list-style-type: none"> • Explain the effects of arithmetic operations on fractions and decimals 	Students demonstrate a thorough understanding when comparing and ordering fractions, decimals, mixed number sand integers.	Students readily compare and order fractions, decimals, mixed numbers and integers.	Students compare and order fractions, decimals, mixed numbers and integers with some errors.	Students have great difficulty comparing and ordering fractions, decimals, mixed numbers and integers.
	Students generate an accurate list of factors prime factors, multiples, and effectively use rules to determine divisibility by 2, 3, 5, 6, 9, and 10 with no errors.	Students generate an accurate list of factors, prime factors, multiples, and effectively use rules to determine divisibility by 2, 3, 5, 6, 9, and 10 with few errors.	Students generate a list of factors, prime factors, multiples, and use rules to determine divisibility by 2, 3, 5, 6, 9, and 10 with some errors.	Students generate a list of factors, prime factors, multiples, and use rules to determine divisibility by 2, 3, 5, 6, 9, and 10 with many errors.
	Students thoroughly explain effects of arithmetic operations on fractions and decimals.	Students adequately explain effects of arithmetic operations on fractions and decimals.	Students partially explain effects of arithmetic operations on fractions and decimals.	Students rarely explain effects of arithmetic operations on fractions and decimals.

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Identify the uses of the commutative and associative properties of addition and multiplication • Use order of operations (i.e., multiplication, division, addition and subtraction) to simplify numeric expressions <p style="text-align: center;">COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> • Multiply and divide decimals • Add, subtract, multiply, and divide fractions • Express an exponent in standard form 	Students appropriately apply the commutative and associative properties of addition and multiplication.	Students adequately identify the uses of the commutative and associative properties of addition and multiplication.	Students identify the uses of the commutative and associative properties of addition and multiplication in some cases.	Students almost never identify the uses of the commutative and associative properties of addition and multiplication in some cases.
	Students properly and accurately apply order of operations to simplify numeric expressions.	Students properly apply order of operations to simplify numeric expressions.	Students apply order of operations to simplify numeric expressions with some errors.	Students apply order of operations to simplify numeric expressions with many errors.
	Students accurately multiply and divide decimals with no errors.	Students accurately multiply and divide decimals with few errors.	Students multiply and divide decimals with some errors.	Students multiply and divide decimals with many errors.
	Students accurately add, subtract, multiply, and divide fractions with no errors.	Students accurately add, subtract, multiply, and divide fractions with few errors.	Students add, subtract, multiply, and divide fractions with some errors.	Students add, subtract, multiply, and divide fractions with many errors.
	Students correctly express an exponent in standard form with no errors.	Students correctly express an exponent in standard form with few errors.	Students express an exponent in standard form with some errors.	Students express an exponent in standard form with many errors.

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Use problem solving strategies to solve and verify the results of problems • Estimate the results of problems involving whole numbers, fractions, and decimals 	Students effectively use problem solving strategies to solve and verify the results of problems.	Students use problem solving strategies to solve and verify the results of problems.	Students misapply problem solving strategies or do not verify the results of problems.	Student work does not demonstrate the use of problem solving strategies.
	Students properly apply estimates to problems involving whole numbers, fractions and decimals.	Students adequately estimate the results of problems involving whole numbers, fractions and decimals.	Students demonstrate minor misconceptions when estimating the results of problems involving whole numbers, fractions, and decimals.	Students show major misconceptions when estimating the results of problems involving whole numbers, fractions, and decimals.

Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 6</p> <p>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</p> <ul style="list-style-type: none"> • Identify relationships between pairs of angles; i.e., adjacent, vertical, complementary, and supplementary • Identify polygons; i.e., triangle, rectangle, square, rhombus, parallelogram, trapezoid, pentagon, hexagon, octagon • Describe the characteristics of a right triangle <p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> • Use ordered pairs to locate a point on a coordinate plane 	<p>Students consistently and correctly identify relationships between pairs of angles.</p> <p>Students consistently identify polygons.</p> <p>Students describe detail the characteristics of a right triangle.</p> <p>Students properly use ordered pairs to locate a point on a coordinate plane and can write an ordered pair from a given point.</p>	<p>Students consistently and correctly identify relationships between pairs of angles with few errors.</p> <p>Students consistently identify polygons.</p> <p>Students adequately describe the characteristics of a right triangle.</p> <p>Students properly use ordered pairs to locate a point on a coordinate plane.</p>	<p>Students correctly identify relationships between pairs of angles with some errors.</p> <p>Students identify polygons some of the time.</p> <p>Students describe some of the characteristics of a right triangle.</p> <p>Students use ordered pairs to locate a point on a coordinate plane some of the time.</p>	<p>Students identify relationships between pairs of angles with many errors.</p> <p>Students rarely identify polygons.</p> <p>Students can identify a right triangle but have difficulty describing the characteristics of a right triangle.</p> <p>Students improperly use ordered pairs to locate a point on a coordinate plane.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> Identify, describe, and model motion geometry; i.e., rotations, reflections, and translations 	<p>Students consistently identify, describe and model motion geometry.</p>	<p>Students consistently identify and describe, motion geometry.</p>	<p>Students identify and describe some examples of motion geometry.</p>	<p>Students have great difficulty recognizing motion geometry.</p>
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> Draw basic geometric figures using appropriate tools; i.e., circle with a compass, triangle and rectangle with a ruler or straight edge 	<p>Students draw combinations of basic geometric figures using appropriate tools.</p>	<p>Students effectively draw basic geometric figures using appropriate tools.</p>	<p>Students draw basic geometric figures without consistently using appropriate tools.</p>	<p>Students show beginning understanding of the use of appropriate tools when drawing basic geometric figures.</p>

Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 6</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> Collect and organize data, select and use an appropriate display; i.e., a frequency table, a line and bar graph <p>PROBABILITY</p> <ul style="list-style-type: none"> Count possible outcomes using lists Use experiments or simulations to determine probabilities Use decimal values and ratios to represent probability 	<p>Students adequately collect and organize data, and select and use an appropriate display.</p> <p>Students accurately count possible outcomes more efficiently using organized lists.</p> <p>Students effectively use experiments and simulations to determine probabilities.</p> <p>Students correctly use decimal values and ratios to represent probability.</p>	<p>Students adequately collect and organize data, and select and use an appropriate display with few omissions.</p> <p>Students accurately count possible outcomes with few omissions using lists.</p> <p>Students effectively use experiments or simulations to determine probabilities.</p> <p>Students correctly use decimal values and ratios to represent probability with few errors.</p>	<p>Students make errors in the collection, organization, and/or display of data.</p> <p>Students miscount possible outcomes due to unorganized lists.</p> <p>Students use experiments or simulations but determine inaccurate probabilities.</p> <p>Students use decimal values and ratios to represent probability with some errors.</p>	<p>Students have great difficulty when collecting, organizing, or displaying data.</p> <p>Students have great difficulty generating lists to count possible outcomes.</p> <p>Students have great difficulty determining probabilities.</p> <p>Students use decimal values and ratios to represent probability with many errors.</p>

NORTH DAKOTA
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>STATISTICAL METHODS</p> <ul style="list-style-type: none"> Calculate the mean, median, mode, and range of a set of data 	<p>Students calculate mean, median, mode and range of a set of data.</p>	<p>Students calculate mean, median, mode and range of a set of data with few errors.</p>	<p>Students calculate mean, median, mode and range of a set of data with some errors.</p>	<p>Students calculate mean, median, mode and range of a set of data with many errors.</p>
<p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> Make predictions based on trends identified in tables and graphs 	<p>Students consistently make reasonable predictions based on trends identified in tables and graphs.</p>	<p>Students frequently make reasonable predictions based on trends identified in tables and graphs.</p>	<p>Students occasionally make reasonable predictions based on trends identified in tables and graphs.</p>	<p>Students identify trends in tables and graphs but cannot make reasonable predictions.</p>

Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 6</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> • Measure to the nearest sixteenth of an inch • Select an appropriate unit of measure; e.g., What unit do you use to measure a person’s height? • Convert unit measurements within the same system (metric and standard) • Distinguish between perimeter, area, surface area, and volume • Estimate the perimeter and area of two-dimensional figures • Estimate the surface area and volume of three-dimensional objects 	<p>Students accurately measure to the nearest sixteenth of an inch.</p> <p>Students consistently select an appropriate unit of measure.</p> <p>Students accurately convert unit measurements within the same system.</p> <p>Students correctly distinguish among perimeter, area, surface area, and volume in problem solving situations.</p>	<p>Students accurately measure to the nearest sixteenth of an inch most of the time.</p> <p>Students select an appropriate unit of measure most of the time.</p> <p>Students convert unit measurements within the same system with few errors.</p> <p>Students correctly distinguish among perimeter, area, surface area, and volume most of the time.</p>	<p>Students have difficulty measuring to the nearest sixteenth of an inch.</p> <p>Students select an appropriate unit of measure some of the time.</p> <p>Students convert unit measurements within the same system with some errors.</p> <p>Students correctly distinguish among perimeter, area, surface area, and volume some of the time.</p>	<p>Students can identify markings for a sixteenth of an inch on a ruler, but have great difficulty using them for measurement.</p> <p>Students select an appropriate unit of measure with great difficulty.</p> <p>Students convert unit measurements within the same system with many errors.</p> <p>Students have difficulty distinguishing among perimeter, area, surface area, and volume.</p>

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> • Select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area, and volume • Use formulas to determine the circumference and area of circles and the perimeter and area of triangles and parallelograms • Use area formulas to determine the surface area of right prisms and square pyramids • Use formulas to determine the volume of rectangular prisms 	<p>Students consistently select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area and volume.</p> <p>Students properly apply formulas and accurately determine the circumference and area of circles, the perimeter and area of triangles and parallelograms, the surface area of right prisms and square pyramids, and the volume of rectangular prisms.</p>	<p>Students select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area and volume most of the time.</p> <p>Students properly apply formulas to determine, with minor computational errors, the circumference and area of circles, the perimeter and area of triangles and parallelograms, the surface area of right prisms and square pyramids, and the volume of rectangular prisms.</p>	<p>Students select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area and volume some of the time.</p> <p>Students attempt to apply formulas to determine the circumference and area of circles, the perimeter and area of triangles and parallelograms, the surface area of right prisms and square pyramids, and the volume of rectangular prisms.</p>	<p>Students have great difficulty selecting appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area and volume.</p> <p>Students misapply formulas to determine the circumference and area of circles, the perimeter and area of triangles and parallelograms, the surface area of right prisms and square pyramids, and the volume of rectangular prisms.</p>

Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 6</p> <p>PATTERNS, RELATIONS, AND FUNCTIONS</p> <ul style="list-style-type: none"> Identify and describe patterns represented by tables, graphs, and sequences <p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> Use a variable to represent an unknown quantity <p>MATHEMATICAL MODELING</p> <ul style="list-style-type: none"> Use representations to solve problems; i.e., tables and numerical sentences <p>RATES OF CHANGE</p> <ul style="list-style-type: none"> Recognize examples of change over time; e.g. growth of a sixth grader from September to May 	<p>Students consistently identify and describe the patterns represented by tables, graphs, and sequences.</p> <p>Students consistently and correctly use a variable to represent an unknown quantity.</p> <p>Student’s use of representations to solve problems demonstrates a deep understanding.</p> <p>Students describe specific examples of change over time.</p>	<p>Students usually identify and describe the patterns represented by tables, graphs, and sequences.</p> <p>Students correctly use a variable to represent an unknown quantity most of the time.</p> <p>Students consistently and correctly use appropriate representations to solve problems.</p> <p>Students consistently recognize examples of change over time.</p>	<p>Students sometimes identify and describe the patterns represented by tables, graphs, and sequences.</p> <p>Students correctly use a variable to represent an unknown quantity some of the time.</p> <p>Students correctly use appropriate representations to solve problems most of the time.</p> <p>Students generally recognize examples of change over time.</p>	<p>Students rarely identify and describe the patterns represented by tables, graphs, and sequences.</p> <p>Students have great difficulty using a variable to represent an unknown quantity.</p> <p>Students use appropriate representations to solve problems some of the time.</p> <p>Students have great difficulty recognizing examples of change over time.</p>