

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

Content standard – A description of what students should know and be able to do within a particular content discipline or subject.

Subtopic - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

Benchmark expectation - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

Achievement standard A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

Advanced Proficient -Demonstrates exemplary understanding and exceeds expected level of performance

Proficient - Demonstrates understanding and meets expected level of performance.

Partially Proficient - Demonstrates an emerging or developing level of understanding and performance.

Novice – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 8</p> <p>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</p> <ul style="list-style-type: none"> • Identify subsets of the real number system; i.e. natural and whole numbers, integers, rational and irrational numbers • Solve real-world problems involving ratio, proportion, and percent • Identify perfect squares 1 to 144 and approximate square roots • Represent large and small numbers using scientific notation 	<p>Students correctly classify numbers as members of subsets of the real number system and describe the characteristics of the subsets.</p> <p>Students properly apply ratio, proportion, and percent when solving real-world problems.</p> <p>Students readily identify perfect squares 1 to 144 and adequately approximate square roots.</p> <p>Students correctly represent large and small numbers using scientific notation with no errors.</p>	<p>Students correctly classify numbers as members of subsets of the real number system.</p> <p>Students accurately solve real-world problems involving ratio, proportion, and percent.</p> <p>Students identify perfect squares 1 to 144 and adequately approximate square roots.</p> <p>Students correctly represent large and small numbers using scientific notation with few errors.</p>	<p>Students incompletely classify numbers as members of subsets of the real number system.</p> <p>Students show beginning understanding in solving real-world problems involving ratio, proportion, and percent.</p> <p>Students identify perfect squares 1 to 144 and can approximate some square roots.</p> <p>Students correctly represent large and small numbers using scientific notation with some errors.</p>	<p>Students have great difficulty when classifying numbers as members of subsets of the real number system.</p> <p>Students have great difficulty when solving real-world problems involving ratio, proportions, and percent.</p> <p>Students identify some of the perfect squares 1 to 144 and have great difficulty approximating square roots.</p> <p>Students correctly represent large and small numbers using scientific notation with many errors.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>OPERATIONS AND THEIR PROPERTIES</p> <ul style="list-style-type: none"> Apply operation properties to simplify computations and solve problems; i.e. commutative, associative, and distributive Apply the order of operations to simplify numeric expressions and solve problems 	<p>Students properly apply operation properties to simplify computations and solve problems.</p>	<p>Students properly apply operation properties to simplify computations and solve problems with few computational errors.</p>	<p>Students apply operation properties to simplify computations and solve problems with few computational or process errors.</p>	<p>Students apply operation properties to simplify computations and solve problems with many errors.</p>
<p>COMPUTATIONAL FLUENCY AND ESTIMATION</p> <ul style="list-style-type: none"> Add, subtract, multiply, and divide integers Select and use an appropriate computational technique; e.g. mental calculation, paper-and-pencil, or technology, to solve problems 	<p>Students properly apply the order of operations to simplify numeric expressions and solve problems with no errors.</p> <p>Students accurately add, subtract, multiply, and divide integers with no errors.</p> <p>Students consistently select and use an appropriate computational technique to solve problems with no computational errors.</p>	<p>Students properly apply the order of operations to simplify numeric expressions and solve problems with few computational errors.</p> <p>Students accurately add, subtract, multiply, and divide integers with few errors.</p> <p>Students consistently select and use an appropriate computational technique to solve problems.</p>	<p>Students misapply the order of operations in simplifying numeric expressions and solving problems.</p> <p>Students accurately add, subtract, multiply, and divide integers with few errors with some errors.</p> <p>Students inconsistently select and use an appropriate computational technique to solve problems.</p>	<p>Students apply the order of operations to simplify numeric expressions and solve problems with many errors.</p> <p>Students accurately add, subtract, multiply, and divide integers with many errors.</p> <p>Students have great difficulty when selecting and using an appropriate computational technique to solve problems.</p>

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Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> Determine when an estimate is sufficient and an exact answer is needed in problem situations 	<p>Students readily and correctly determine and justify when an estimate is sufficient and an exact answer is needed in problem situations.</p>	<p>Students readily and correctly determine when an estimate is sufficient and an exact answer is needed in problem situations.</p>	<p>Students sometimes determine when an estimate is sufficient and an exact answer is needed in problem situations.</p>	<p>Students have great difficulty in determining when an estimate is sufficient and an exact answer is needed in problem situations.</p>

Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 8</p> <p>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</p> <ul style="list-style-type: none"> • Make observations about relationships between two- and three-dimensional figures; e.g., a cube is made with six squares • Classify quadrilaterals based on side lengths, angle measures, and sets of parallel sides 	<p>Students' observations about relationships between two- and three-dimensional figures show deep understanding of the concept.</p> <p>Students show appropriate connections when classifying quadrilaterals based on side length, angle measures, and sets of parallel sides.</p>	<p>Students make relevant observations about relationships between two- and three-dimensional figures.</p> <p>Students classify quadrilaterals based on side length, angle measures, and sets of parallel sides.</p>	<p>Students make limited observations about relationships between two- and three-dimensional figures.</p> <p>Students classify some quadrilaterals based on side length, angle measures, and sets of parallel sides.</p>	<p>Students show beginning understanding of the concept about relationships between two- and three-dimensional figures.</p> <p>Students have great difficulty when classifying quadrilaterals based on side length, angle measures, and sets of parallel sides.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> • Identify the angles formed and the relationships between the angles when parallel lines are intersected by a transversal • Apply the Pythagorean Theorem to problems involving right triangles <p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> • Represent shapes using coordinate geometry 	Students consistently identify the angles formed and the relationships between the angles when parallel lines are intersected by a transversal.	Students consistently, and with minor errors, identify the angles formed and the relationships between the angles when parallel lines are intersected by a transversal.	Students identify with some errors the angles formed and the relationships between the angles when parallel lines are intersected by a transversal.	Students identify the angles formed and the relationships between the angles but cannot recognize when parallel lines are intersected by a transversal.
	Students properly apply the Pythagorean Theorem to problems involving right triangles.	Students properly apply the Pythagorean Theorem to problems involving right triangles with few errors in computation.	Students misapply the Pythagorean Theorem to problems involving right triangles.	Students show little understanding of the Pythagorean Theorem.
	Students represent shapes using coordinate geometry.	Students adequately represent shapes using coordinate geometry.	Students represent some shapes using coordinate geometry.	Students have great difficulty when representing some shapes using coordinate geometry.

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> • Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations • Use scale, proportion, and congruency to solve problems involving similar figures 	<p>Students draw the results of a combination of transformations in the coordinate plane.</p> <p>Students appropriately use scale, proportion, and congruency to solve problems involving similar figures.</p>	<p>Students adequately draw the results of a combination of transformations in the coordinate plane.</p> <p>Students appropriately use scale, proportion, and congruency to solve problems involving similar figures with minor errors.</p>	<p>Students draw the results of some combinations of transformations in the coordinate plane.</p> <p>Students use scale, proportion, and congruency to solve problems involving similar figures with some errors.</p>	<p>Students have great difficulty drawing any combination of transformations in the coordinate plane.</p> <p>Students show beginning understanding of scale, proportion, and congruency involving similar figures.</p>
<p>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</p> <ul style="list-style-type: none"> • Use two-dimensional representations of three-dimensional objects to visualize and solve problems; e.g., those involving surface area and volume 	<p>Students appropriately use two-dimensional representations of three-dimensional objects to visualize and solve problems.</p>	<p>Students appropriately use two-dimensional representations of three-dimensional objects to visualize and solve problems with minor errors in computation.</p>	<p>Students use two-dimensional representations of three-dimensional objects to visualize and solve problems without complete understanding.</p>	<p>Students have great difficulty visualizing representations of three-dimensional objects.</p>

Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 8</p> <p>DATA COLLECTION, DISPLAY, AND INTERPRETATION</p> <ul style="list-style-type: none"> Formulate a question and select a random or representative sample Collect, organize, and display data using scatter and stem-and-leaf plot <p>PROBABILITY</p> <ul style="list-style-type: none"> Determine possible outcomes using organized lists, tree diagrams, Venn diagrams, factorials, and the basic counting principle Distinguish between experimental and theoretical probability; i.e., the results of an experiment may not match the theoretical probability 	<p>Students formulate a relevant question and demonstrate insight when selecting a random or representative sample.</p> <p>Students collect, organize, and display data using scatter and stem-and-leaf plots.</p> <p>Students accurately determine possible outcomes using organized lists, tree diagrams, Venn diagrams, factorials, and the basic counting principle.</p> <p>Students demonstrate insight when distinguishing between experimental and theoretical probability.</p>	<p>Students formulate a relevant question and select a random or representative sample.</p> <p>Students collect, organize, and display data using scatter and stem-and-leaf plots with few omissions.</p> <p>Students determine possible outcomes, with few errors, using organized lists, tree diagrams, Venn diagrams, factorials, and the basic counting principle.</p> <p>Students properly distinguish between experimental and theoretical probability.</p>	<p>Students show misconceptions when formulating a question and selecting a random or representative sample.</p> <p>Students collect, organize, and display data using scatter or stem-and-leaf plots with some omissions.</p> <p>Students determine possible outcomes, with some errors, using organized lists, tree diagrams, Venn diagrams, factorials, and the basic counting principle.</p> <p>Students have beginning understanding between experimental and theoretical probability.</p>	<p>Students have great difficulty formulating a question, selecting a random or representative sample.</p> <p>Students collect data and show a beginning understanding of organizing and displaying data.</p> <p>Students determine possible outcomes, with many errors, using organized lists, tree diagrams, Venn diagrams, factorials, and the basic counting principle.</p> <p>Students cannot distinguish between experimental and theoretical probability.</p>

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Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>STATISTICAL METHODS</p> <ul style="list-style-type: none"> • Calculate and compare the measures of central tendency (i.e., mean, median, mode) and spread (i.e., range) • Identify an outlier within a set of data and discuss its effects on the measures of central tendency and spread 	<p>Students accurately calculate and demonstrate insight when making a comparison of the measures of central tendency and spread.</p>	<p>Students calculate and make a reasonable comparison of the measures of central tendency and spread.</p>	<p>Students calculate, but have difficulty making a reasonable comparison of the measures of central tendency and spread.</p>	<p>Students miscalculate and have great difficulty when comparing the measures of central tendency and spread.</p>
<p>PREDICTIONS, DATA ANALYSIS AND INFERENCES</p> <ul style="list-style-type: none"> • Make inferences based on analysis of data and interpretation of graphs 	<p>Students properly identify an outlier within a set of data and thoroughly explain its effects on the measures of central tendency and spread.</p>	<p>Students properly identify an outlier within a set of data and adequately explain its effects on the measures of central tendency and spread.</p>	<p>Students identify an outlier within a set of data and vaguely explain its effects on the measures of central tendency and spread.</p>	<p>Students identify an outlier within a set of data and cannot explain its effects on the measures of central tendency and spread.</p>
<p>Students make creative inferences based on analysis of data and interpretations of graphs.</p>	<p>Students make reasonable inferences based on analysis of data and interpretations of graphs.</p>	<p>Students make inferences without justification based on analysis of data and interpretations of graphs.</p>	<p>Students make unreasonable inferences based on analysis of data and interpretations of graphs.</p>	<p>Students make unreasonable inferences based on analysis of data and interpretations of graphs.</p>

Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 8</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> Select an appropriate degree of precision when using measurements for calculations Compare unit measurements between systems; e.g., a yard is almost a meter <p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> Use formulas to determine the surface area and volume of right cones and spheres 	<p>Students consistently determine an appropriate degree of precision when using measurements for calculations in problem solving situations.</p> <p>Students make specific comparisons of unit measurements between systems.</p> <p>Students properly apply formulas to accurately determine the surface area and volume of right cones and spheres.</p>	<p>Students usually determine an appropriate degree of precision when using measurements for calculations in problem solving situations.</p> <p>Students make general comparisons of unit measurements between systems.</p> <p>Students properly apply formulas to determine the surface area and volume of right cones and spheres with minor computational errors.</p>	<p>Students sometimes determine an appropriate degree of precision when using measurements for calculations.</p> <p>Students make statements that indicate the basic relationship between systems of measurement.</p> <p>Students make procedural errors when applying formulas to determine the surface area and volume of right cones and spheres.</p>	<p>Students have great difficulty determining an appropriate degree of precision when using measurements for calculations.</p> <p>Students make statements they do not indicate the correct relationship between systems of measurement.</p> <p>Students apply formulas to determine the surface area and volume of right cones and spheres with substantial errors.</p>

Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grade 8</p> <p>PATTERNS, RELATIONS, AND FUNCTIONS</p> <ul style="list-style-type: none"> Extend numerical patterns; e.g., Pascal’s triangle and the Fibonacci sequence <p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> Use variables, expressions and equations to represent problem situations <ul style="list-style-type: none"> Apply the order of operations and the commutative, associative, and distributive properties to evaluate numeric expressions and simplify algebraic expressions 	<p>Students accurately extend numerical patterns.</p> <p>Students appropriately use variables, expressions, and equations to represent problem situations.</p> <p>Students properly apply the order of operations and the commutative, associative, and distributive properties to accurately evaluate numeric expressions and simplify algebraic expressions.</p>	<p>Students extend numerical patterns with minor errors.</p> <p>Students appropriately use variables, expressions, and equations to represent problem situations most of the time.</p> <p>Students properly apply the order of operations and the commutative, associative, and distributive properties to evaluate numeric expressions and simplify algebraic expressions with minor computational errors.</p>	<p>Students extend simple numerical patterns with some errors.</p> <p>Students appropriately use variables, expressions, and equations to represent problem situations some of the time.</p> <p>Students apply the order of operations and the commutative, associative, and distributive properties to evaluate numeric expressions and simplify algebraic expressions with some procedural errors.</p>	<p>Students accurately extend numerical patterns.</p> <p>Students have great difficulty using variables, expressions, and equations to represent problem situations.</p> <p>Students show a beginning understanding of the order of operations and the commutative, associative, and distributive properties in evaluating numeric expressions and simplifying algebraic expressions.</p>

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Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>MATHEMATICAL MODELING</p> <ul style="list-style-type: none"> Students apply inverse operations and the properties of equality to solve multi-step equations and inequalities in one variable 	<p>Students properly apply inverse operations and the properties of equality to accurately solve multi-step equations and inequalities in one variable.</p> <p>Students demonstrate a deeper understanding of multi-step equations and inequalities by representing a problem situation in more than one way.</p>	<p>Students properly apply inverse operations and the properties of equality to solve multi-step equations and inequalities in one variable with minor computational errors.</p> <p>Students appropriately represent problem situations by writing multi-step equations and inequalities.</p>	<p>Students apply inverse operations and the properties of equality to solve multi-step equations and inequalities in one variable with some procedural errors.</p> <p>Students represent problem situations by writing multi-step equations and inequalities some of the time</p>	<p>Students understand inverse operations but have great difficulty solving multi-step equations and inequalities in one variable</p> <p>Students have great difficulty representing problem situations with multi-step equations and inequalities.</p>
<p>RATES OF CHANGE</p> <ul style="list-style-type: none"> Solve problems involving rates; i.e., distance equals rate times time 	<p>Students accurately solve problems involving rates.</p>	<p>Students solve problems involving rates with minor computational errors.</p>	<p>Students accurately solve problems involving rates with some procedural errors.</p>	<p>Students solve problems involving rates with substantial errors.</p>