

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

**Content standard** – A description of what students should know and be able to do within a particular content discipline or subject.

**Subtopic** - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

**Benchmark expectation** - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

**Achievement standard** A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

**Advanced Proficient** -Demonstrates exemplary understanding and exceeds expected level of performance

**Proficient** - Demonstrates understanding and meets expected level of performance.

**Partially Proficient** - Demonstrates an emerging or developing level of understanding and performance.

**Novice** – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)







## Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 9-10</p> <p><b>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</b></p> <ul style="list-style-type: none"> <li>• Identify the properties and attributes of two- and three-dimensional objects that distinguish one from another; e.g., a cylinder has two parallel circular bases</li>   <li>• Determine congruence and similarity among geometric objects</li> </ul>	<p>Students demonstrate an extensive understanding of the properties and attributes of two- and three-dimensional objects that distinguish one from another.</p> <p>Students draw accurate conclusions that are not immediately obvious about congruence and similarity among geometric objects.</p>	<p>Students demonstrate a substantial understanding of the properties and attributes of two- and three-dimensional objects that distinguish one from another.</p> <p>Students draw accurate conclusions about congruence and similarity among geometric objects.</p>	<p>Students demonstrate a general understanding of the properties and attributes of two- and three-dimensional objects that distinguish one from another.</p> <p>Students draw conclusions about congruence and similarity among geometric objects without complete understanding.</p>	<p>Students demonstrate a limited understanding of the properties and attributes of two- and three-dimensional objects that distinguish one from another.</p> <p>Students draw invalid or no conclusions about congruence and similarity among geometric objects.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Use trigonometric relationships and the Pythagorean Theorem to determine side lengths and angle measures in right triangles</li>   <li>• Using given information, establish the validity of a conjecture using a two-column or paragraph proof</li> </ul>	<p>Students correctly use trigonometric relationships and the Pythagorean Theorem to accurately determine side lengths and angle measures in right triangles.</p> <p>Using given information, students thoroughly and efficiently establish the validity of a conjecture using a two-column or paragraph proof.</p>	<p>Students correctly use trigonometric relationships and the Pythagorean Theorem to determine side lengths and angle measures in right triangles, with minor errors.</p> <p>Using given information, students adequately establish the validity of a conjecture using a two-column or paragraph proof.</p>	<p>Students attempt to use trigonometric relationships and the Pythagorean Theorem to determine side lengths and angle measures in right triangles, with significant errors.</p> <p>Using given information, students omit important details, facts, or concepts when establishing the validity of a conjecture using a two-column or paragraph proof.</p>	<p>Students demonstrate no understanding of trigonometric relationships and the Pythagorean Theorem.</p> <p>Students fail to demonstrate an understanding of the two-column or paragraph proof process.</p>
<p>COORDINATE GEOMETRY</p> <ul style="list-style-type: none"> <li>• Use Cartesian coordinates to determine distance, midpoint, and slope</li> </ul>	<p>Students use Cartesian coordinates to correctly determine distance, midpoint, and slope.</p>	<p>Students use Cartesian coordinates to determine distance, midpoint, and slope, with minor arithmetic errors.</p>	<p>Students use Cartesian coordinates to determine distance, midpoint, and slope, with critical errors.</p>	<p>Students are unable to use Cartesian coordinates to determine distance, midpoint, and slope.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
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<ul style="list-style-type: none"> <li>Use distance, midpoint, and slope to establish relationships between points, lines, and plane figures in the Cartesian coordinate system; e.g., determine whether a triangle is scalene, isosceles, or equilateral given the coordinates of its vertices</li> </ul>	<p>Students use distance, midpoint, and slope to demonstrate an extensive understanding of the relationships between points, lines, and plane figures in the Cartesian coordinate system.</p>	<p>Students use distance, midpoint, and slope to demonstrate a substantial understanding of the relationships between points, lines, and plane figures in the Cartesian coordinate system.</p>	<p>Students use distance, midpoint, and slope to demonstrate a general understanding of the relationships between points, lines, and plane figures in the Cartesian coordinate system.</p>	<p>Students use distance, midpoint, and slope to demonstrate a limited understanding of the relationships between points, lines, and plane figures in the Cartesian coordinate system.</p>
<p>TRANSFORMATION AND SYMMETRY</p> <ul style="list-style-type: none"> <li>Identify and perform transformations (i.e., translations, reflections, rotations, and dilations) of objects in the plane using sketches and coordinates</li> </ul>	<p>Students describe coordinate transformations in detail and perform them with a high degree of accuracy.</p>	<p>Students perform coordinate transformations with a reasonable degree of accuracy and use formal terminology to identify them.</p>	<p>Students identify coordinate transformations or perform them with some accuracy.</p>	<p>Students are unable to identify or perform coordinate transformations.</p>
<ul style="list-style-type: none"> <li>Analyze the effects of combining basic transformations in a plane; e.g., two reflections over parallel lines results in a translation</li> </ul>	<p>Students describe, in comprehensive detail, the results of combining basic transformations in a plane.</p>	<p>Students describe, in adequate detail, the results of combining basic transformations in a plane.</p>	<p>Students describe, in limited detail, the results of combining basic transformations in a plane.</p>	<p>Students are unable to describe the results of combining basic transformations in a plane.</p>

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
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<p><b>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</b></p> <ul style="list-style-type: none"> <li>• Construct plane figures using traditional and/or technological tools; i.e., congruent segments, congruent angles, angle and segment bisectors, perpendicular and parallel lines</li>   <li>• Given a two-dimensional image of a three-dimensional object, recognize images of the same object shown from different perspectives</li>   <li>• Use geometric models to gain insights into, and to answer questions in, other areas of mathematics, other disciplines, and other areas of interest; e.g., art and architecture</li> </ul>	<p>Students use appropriate methods to construct plane figures with a high degree of accuracy using traditional and/or technological tools.</p> <p>Given a two-dimensional image of a three-dimensional object, students correctly recognize images of the same object shown from different perspectives and provide additional insight.</p> <p>Students use geometric models to find creative solutions to practical problems.</p>	<p>Students use appropriate methods to construct plane figures with reasonable accuracy using traditional and/or technological tools.</p> <p>Given a two-dimensional image of a three-dimensional object, students correctly recognize images of the same object shown from different perspectives.</p> <p>Students use geometric models to find reasonable solutions to practical problems.</p>	<p>Students construct plane figures with minimal accuracy using traditional and/or technological tools.</p> <p>Given a two-dimensional image of a three-dimensional object, students show some recognition of images of the same object shown from different perspectives.</p> <p>Students attempt to use geometric models to find solutions to practical problems.</p>	<p>Students show little ability to construct plane figures using traditional and/or technological tools.</p> <p>Given a two-dimensional image of a three-dimensional object, students are unable to recognize images of the same object shown from different perspectives.</p> <p>Students are unable to use geometric models to find solutions to practical problems.</p>

## Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 9-10</p> <p><b>DATA COLLECTION, DISPLAY, AND INTERPRETATION</b></p> <ul style="list-style-type: none"> <li>Construct appropriate displays of given data; i.e., circle graphs, bar graphs, histograms, stem-and-leaf plots, box-and-whisker plots, and scatter plots</li> <li>Interpret a given visual representation (i.e., circle graphs, bar graphs, histograms, stem-and-leaf plots, box-and-whisker plots, and scatter plots) of a set of data</li> <li>Explain the role of the variable, sample, population, and randomization in a well-designed study</li> </ul> <p><b>PROBABILITY</b></p> <ul style="list-style-type: none"> <li>Determine the number of possible outcomes for a given event, using appropriate counting techniques; e.g., fundamental counting principle, factorials, combinations, permutations</li> </ul>	<p>Students appropriately display a given set of data in a variety of ways.</p> <p>Students provide a thorough and insightful interpretation of a given visual representation for a data set.</p> <p>Students precisely explain the role of the variable, sample, population, and randomization in a well-designed study.</p> <p>Students determine the correct number of possible outcomes for an event, demonstrating a clear understanding of the process.</p>	<p>Students appropriately display a given set of data.</p> <p>Students provide a reasonable interpretation of a given visual representation for a data set.</p> <p>Students adequately explain the role of the variable, sample, population, and randomization in a well-designed study.</p> <p>Students determine the correct number of possible outcomes for an event.</p>	<p>Students appropriately display a given set of data, with some inaccuracies.</p> <p>Students provide an incomplete interpretation of a given visual representation for a data set.</p> <p>Students explain, without complete understanding, the role of the variable, sample, population, or randomization in a well-designed study.</p> <p>Students make a reasonable attempt to determine the correct number of possible outcomes for an event.</p>	<p>Students inappropriately display a given set of data.</p> <p>Students provide a vague interpretation of a given visual representation for a data set.</p> <p>Students are unable to explain the role of the variable, sample, population, and randomization in a well-designed study.</p> <p>Students demonstrate a lack of understanding in attempting to determine the correct number of possible outcomes for an event.</p>

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Calculate experimental and theoretical probabilities with and without replacement</li> <li>• Calculate probabilities of compound events using addition and multiplication rules</li> </ul>	Students accurately determine experimental and theoretical probabilities with and without replacement, providing thoughtful analysis.	Students demonstrate competence in determining experimental and theoretical probabilities with and without replacement, with minor arithmetic errors.	Students demonstrate limited understanding in determining experimental or theoretical probabilities with and without replacement.	Students demonstrate no understanding in determining experimental and theoretical probabilities with and without replacement.
	Students accurately calculate probabilities of compound events, providing insight into the use of addition and multiplication rules.	Students correctly use addition and multiplication rules to calculate probabilities of compound events, with minor computational errors allowed.	Students incorrectly apply addition and multiplication rules to calculate probabilities of compound events.	Students demonstrate no understanding of addition and multiplication rules when calculating probabilities of compound events.
<b>STATISTICAL METHODS</b> <ul style="list-style-type: none"> <li>• Calculate measures of central tendency and spread; i.e., mean, median, mode, range, and quartiles</li> <li>• Discuss relationships among measures of central tendency and spread; i.e., mean, median, mode, range, and quartiles</li> </ul>	Students calculate, with no errors, measures of central tendency and spread.	Students calculate, with few errors, all listed measures of central tendency and spread.	Students calculate some measures of central tendency or spread.	Students are unable to calculate measures of central tendency and spread.
	Students demonstrate exceptional understanding of relationships among measures of central tendency and spread.	Students demonstrate substantial understanding of relationships among measures of central tendency and spread.	Students demonstrate a general understanding of relationships among measures of central tendency or spread.	Students demonstrate little or no understanding of relationships among measures of central tendency or spread.



## Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 9-10</p> <p><b>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</b></p> <ul style="list-style-type: none"> <li>• Select appropriate units and scales for problem situations involving measurement</li>   <li>• Describe the effects of scalar change on the area and volume of a figure; e.g., the effect of doubling one or more edges of a solid on its surface area and volume</li>   <li>• Use approximations to compare the standard and metric systems of measurement; e.g., a five-kilometer race is about three miles long</li> </ul>	<p>Students select and defend appropriate units and scales for problem situations involving measurement.</p> <p>Students provide an accurate and in-depth description of the effects of scalar change on area and volume.</p> <p>Students provide reasonable estimates to conversions between the standard and metric systems of measurement, justifying their reasoning.</p>	<p>Students select appropriate units and scales for problem situations involving measurement.</p> <p>Students accurately describe the effects of scalar change on area and volume.</p> <p>Students provide reasonable estimates to conversions between the standard and metric systems of measurement.</p>	<p>Students select inefficient units and scales for problem situations involving measurement.</p> <p>Students accurately describe the effects of scalar change on area or volume.</p> <p>Students provide poor estimates to conversions between the standard and metric systems of measurement.</p>	<p>Students select inappropriate units or scales for problem situations involving measurement.</p> <p>Students are unable to accurately describe the effects of scalar change on area and volume.</p> <p>Students provide unreasonable estimates to conversions between the standard and metric systems of measurement.</p>

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>Given a conversion factor, convert between standard and metric measurements</li> </ul>	<p>Given a conversion factor, students accurately convert between standard and metric measurements.</p>	<p>Given a conversion factor, students convert between standard and metric measurements, with minor computational errors allowed.</p>	<p>Given a conversion factor, students convert between standard and metric measurements, with significant errors.</p>	<p>Given a conversion factor, students are unable to convert between standard and metric measurements.</p>
<p><b>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</b></p> <ul style="list-style-type: none"> <li>Use methods necessary to achieve a specified degree of precision and accuracy (i.e., appropriate number of significant digits) in measurement situations</li> </ul>	<p>Students correctly use and defend methods necessary to achieve a specified degree of precision and accuracy in a measurement situation.</p>	<p>Students correctly use methods necessary to achieve a specified degree of precision and accuracy in a measurement situation.</p>	<p>Students show limited understanding about precision and accuracy in a measurement situation.</p>	<p>Students show a lack of understanding about precision and accuracy in a measurement situation.</p>
<ul style="list-style-type: none"> <li>Employ estimation techniques to evaluate reasonableness of results in measurement situations</li> </ul>	<p>Students appropriately employ estimation techniques to evaluate reasonableness of results in measurement situations, using an inventive approach.</p>	<p>Students appropriately employ estimation techniques to evaluate reasonableness of results in measurement situations.</p>	<p>Students employ estimation techniques to evaluate reasonableness of results in measurement situations, overlooking critical details.</p>	<p>Students are unable to assess reasonableness of results in measurement situations.</p>

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Use unit analysis to track units during computations</li>   <li>• Given a formula list, compute the area of a regular polygon</li>   <li>• Given a formula list, compute the surface area and volume of a right prism, right cylinder, right pyramid, right cone, and sphere</li>   <li>• Apply indirect measurement techniques to solve problems involving irregular shapes or inaccessible objects; e.g., calculate the distance across a lake, triangulate an irregular region to find its approximate area</li> </ul>	<p>Students use and explain unit analysis to correctly track units during computations.</p> <p>Given a formula list, students correctly compute the area of a regular polygon, the surface area of a solid, and the volume of a solid.</p> <p>Students demonstrate exceptional creativity in applying indirect measurement techniques to solve problems involving irregular shapes or inaccessible objects with reasonable accuracy.</p>	<p>Students use unit analysis to correctly track units during computations.</p> <p>Given a formula list, students correctly compute the area of a regular polygon, the surface area of a solid, and the volume of a solid, allowing for minor errors.</p> <p>Students apply indirect measurement techniques to solve problems involving irregular shapes or inaccessible objects with reasonable accuracy.</p>	<p>Students use unit analysis to track units during computations, with minor errors.</p> <p>Given a formula list, students compute the area of a regular polygon, the surface area of a solid, and the volume of a solid, with significant errors.</p> <p>Students apply inappropriate measurement techniques when attempting to solve problems involving irregular shapes or inaccessible objects.</p>	<p>Students incorrectly track units during computations.</p> <p>Given a formula list, students are unable to compute the area of a regular polygon, the surface area of a solid, or the volume of a solid.</p> <p>Students are unable to solve problems involving irregular shapes or inaccessible objects.</p>

## Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Grades 9-10</p> <p><b>PATTERNS, RELATIONS, AND FUNCTIONS</b></p> <ul style="list-style-type: none"> <li>• Given the explicit and/or the recursive definition of a sequence, generate a specified number of terms</li>   <li>• Express relations and functions using a variety of representations; i.e., numeric, graphic, symbolic, and verbal</li>   <li>• Determine whether a relation is a function by examining various representations of the relation; e.g., table, graph, equation, set of ordered pairs</li> </ul>	<p>Given an explicit and/or a recursive definition of a sequence, students accurately generate the specified number of terms and provide insight regarding long-term behavior of the function.</p> <p>Given one representation of a relation, students correctly provide three additional representations.</p> <p>Students correctly determine whether a relation is a function and provide valid justification in comprehensive detail.</p>	<p>Given an explicit and/or a recursive definition of a sequence, students accurately generate the specified number of terms.</p> <p>Given one representation of a relation, students correctly provide two additional representations.</p> <p>Students correctly determine whether a relation is a function and provide valid justification.</p>	<p>Given an explicit and/or a recursive definition of a sequence, students generate the specified number of terms, with minor errors.</p> <p>Given one representation of a relation, students correctly provide one additional representation.</p> <p>Students correctly determine whether a relation is a function and provide vague justification.</p>	<p>Given an explicit and/or a recursive definition of a sequence, students generate the specified number of terms, with major errors.</p> <p>Given one representation of a relation, students are unable to provide an additional representation.</p> <p>Students are unable to correctly determine whether a relation is a function.</p>

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Perform the operations of addition, subtraction, multiplication, and division on algebraic functions; e.g., Given <math>f(x) = 2x</math> and <math>g(x) = 5x - 7</math>, find <math>f(x) + g(x)</math></li>   <li>• Identify the independent variable, dependent variable, domain, and range of a function</li>   <li>• Draw graphs of linear and quadratic functions using paper and pencil, labeling key features; e.g., graph a line and label its x-intercept and y-intercept, graph a parabola and label its vertex and one point on each side of the vertex</li> </ul>	Students correctly perform the four basic operations on algebraic functions.	Students perform, with few computational errors, the four basic operations on algebraic functions.	Students perform, with procedural errors, the four basic operations on algebraic functions.	Students are unable to perform the four basic operations on algebraic functions.
	Students demonstrate a thorough understanding of the independent variable, dependent variable, domain, and range of a function.	Students correctly identify the independent variable, dependent variable, domain, and range of a function.	Students demonstrate limited understanding of the independent variable, dependent variable, domain, and range of a function.	Students demonstrate inadequate understanding of the independent variable, dependent variable, domain, and range of a function.
	Students correctly graph linear and quadratic functions, providing extensive detail.	Students correctly graph linear and quadratic functions, labeling some key features.	Students make a reasonable attempt to graph linear and quadratic functions.	Students are unable to graph linear and quadratic functions.
<p>NUMERIC AND ALGEBRAIC REPRESENTATIONS</p> <ul style="list-style-type: none"> <li>• Develop algebraic expressions or equations involving one or two variables to represent relationships found in various contexts (time and distance problems, mixture problems, etc.); e.g., given a verbal statement, write an equivalent algebraic expression or equation</li> </ul>	Students correctly represent a relationship as one or more algebraic expression(s) or equation(s) and define the variable(s).	Students correctly represent a relationship as one or more algebraic expression(s) or equation(s).	Students make a reasonable attempt to represent a relationship as one or more algebraic expression(s) or equation(s).	Students are unable to represent a relationship as one or more algebraic expression(s) or equation(s).

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Manipulate algebraic expressions and equations using properties of real numbers</li>   <li>• Solve linear equations and inequalities, systems of two linear equations or inequalities, and quadratic equations having rational solutions</li>   <li>• Solve a literal equation for a specified variable; e.g., solve <math>I = prt</math> for <math>r</math>, or solve <math>7n + p = t</math> for <math>n</math></li> </ul>	Students correctly manipulate algebraic expressions and equations using properties of real numbers.	Students manipulate, with few computational errors, algebraic expressions and equations using properties of real numbers.	Students manipulate, with procedural errors, algebraic expressions and equations using properties of real numbers.	Students are unable to manipulate algebraic expressions and equations using properties of real numbers.
	Students correctly solve equations, systems, and inequalities.	Students solve, with few computational errors, equations, systems, and inequalities.	Students solve, with procedural errors, equations, systems, and inequalities.	Students are unable to solve equations, systems, and inequalities.
	Students correctly solve a literal equation for a specified variable, including additional information such as equivalent forms or unit analysis.	Students correctly solve a literal equation for a specified variable.	Students solve, with minor errors, a literal equation for a specified variable.	Students are unable to solve a literal equation for a specified variable.

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
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<p><b>MATHEMATICAL MODELING</b></p> <ul style="list-style-type: none"> <li>• Use essential quantitative relationships in a situation to determine whether the relationship can be modeled by a linear function; e.g., simple interest is linear, compound interest is not linear</li>   <li>• Graphically represent the solution or solutions to an equation, inequality, or system</li>   <li>• Interpret a graphical representation of a real-world situation</li>   <li>• Draw conclusions about a situation being modeled</li> </ul>	<p>Students demonstrate extensive understanding about quantitative relationships in a situation and correctly determine whether the relationship can be modeled by a linear function.</p> <p>Students correctly graph and interpret the solution or solutions to an equation, inequality, or system.</p> <p>Students interpret, in extensive detail, a graphical representation of a real-world situation.</p> <p>Given a situation being modeled, students draw conclusions that are not immediately obvious.</p>	<p>Students demonstrate substantial understanding about quantitative relationships in a situation and correctly determine whether the relationship can be modeled by a linear function.</p> <p>Students correctly graph the solution or solutions to an equation, inequality, or system.</p> <p>Students interpret, in adequate detail, a graphical representation of a real-world situation.</p> <p>Students draw reasonable conclusions about a situation being modeled.</p>	<p>Students demonstrate general understanding about quantitative relationships in a situation and correctly determine whether the relationship can be modeled by a linear function.</p> <p>Students make a reasonable attempt to graph the solution or solutions to an equation, inequality, or system.</p> <p>Students interpret, in limited detail, a graphical representation of a real-world situation.</p> <p>Students demonstrate limited understanding when drawing conclusions about a situation being modeled.</p>	<p>Students demonstrate little or no understanding about quantitative relationships in a situation and are unable to determine whether the relationship can be modeled by a linear function.</p> <p>Students are unable to graph the solution or solutions to an equation, inequality, or system.</p> <p>Students are unable to interpret a graphical representation of a real-world situation.</p> <p>Students demonstrate a lack of understanding when drawing conclusions about a situation being modeled.</p>

NORTH DAKOTA  
DRAFT—MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADES K-12—DRAFT

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
RATES OF CHANGE <ul style="list-style-type: none"> <li>• Approximate and interpret rates of change from graphical and numerical data</li> </ul>	Given graphical and numerical data, students demonstrate extensive understanding when providing approximations and interpretations for rates of change.	Given graphical and numerical data, students provide reasonable approximations and meaningful interpretations for rates of change.	Given graphical and numerical data, students demonstrate limited understanding when providing approximations and interpretations for rates of change.	Given graphical and numerical data, students demonstrate a lack of understanding when providing approximations and interpretations for rates of change.