

A high-quality mathematics program is essential for all students. It provides a foundation for intelligent and precise thinking. Mathematics should also provide every student with the opportunity to choose among a full range of future career paths and to contribute to society as an informed citizen. In order to be a responsible and productive member of today’s technological society a student needs to have a broad, connected and useful knowledge of mathematics.

This revision of the North Dakota Mathematics Content and Achievement Standards is intended to provide a framework for the mathematical skills and knowledge that students in grades K-12 are expected to attain. Based on the 1999 North Dakota Mathematics Content Standards, this document was revised to include standards and benchmarks for each grade as well as defining levels of achievement. Additional references include the NCTM *Principles and Standards for School Mathematics* and standards documents from other states.

The standards in this document provide clear, concise, and measurable mathematical expectations for all students. The standards set targets and expectations for what teachers need to teach and students need to learn by the end of each grade level. Parents, community members, and state and local policy makers play an integral part in helping students attain these expectations. This document is a useful resource for school districts as they align their mathematics curriculum to meet mandatory state assessments that are based on these content and achievement standards.

The standards focus on essential content for all students. Maintaining high expectations for all students is a component of equity in education. “All students” include those with diverse cultural backgrounds, limited English proficiency, or disabilities; those who are gifted and talented; and those from advantaged or disadvantaged socioeconomic backgrounds. It includes students, who after high school, choose to enter the workforce, pursue technical career preparation, or attend college.

This document is organized around a core of fundamental mathematics standards for all students in the State of North Dakota. Benchmarks for each standard were written for every grade level, kindergarten through grade eight, grade nine-ten, and grade eleven-twelve. These benchmarks reflect what every student should know and be able to do at the end of each specified grade level. The document is organized in the following way:

**Content standard** – A description of what students should know and be able to do within a particular content discipline or subject.

**Subtopic** - A category within a content standard that aids in the organization of benchmark expectations and that may carry across grade levels.

**Benchmark expectation** - A translation of a standard into what students should know and be able to do at specified grade levels. It is a statement that clearly specifies and itemizes the content of a standard at a specific grade level. When found within a benchmark, “i.e.” means “these specific items,” and “e.g.” means “for example.”

**Achievement standard** A description of what a student knows and can do to demonstrate a level of proficiency on a content standard. Descriptors for achievement are set at four levels and are defined as follows:

**Advanced Proficient** -Demonstrates exemplary understanding and exceeds expected level of performance

**Proficient** - Demonstrates understanding and meets expected level of performance.

**Partially Proficient** - Demonstrates an emerging or developing level of understanding and performance.

**Novice** – Attempt made; however, lack of understanding and performance evident.

The standards in this document are not intended to encompass the entire curriculum for a given grade level. School districts are encouraged to go beyond these standards to help ensure that all students experience a rich mathematics curriculum. In addition, a mathematics education requires more than high quality content. Mathematics instruction should reflect what both educational research and best practice have to say about the teaching and learning of mathematics. It should include hands-on experiences, use of manipulatives, student inquiry, and integrated and regular use of appropriate technologies. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving used in science, engineering, business and industry, government, and practical affairs. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skill should be the shared responsibility of teachers of all disciplines.

Maintaining high expectations for mathematics achievement requires students to go beyond listening to lectures and working textbook problems. Students should spend time on a regular basis generating, discussing and writing about mathematical ideas. The process of mathematics is just as important as the content. The following six goals, which address the processes of mathematics, are intended to accompany the content standards and are embedded within the benchmarks:

- Students will become mathematical problem solvers.
- Students will be able to reason mathematically.
- Students will be confident in their mathematical abilities.
- Students will be able to communicate mathematically.

- Students will be able to make mathematical connections.
- Students will be able to use appropriate technology.

The chart on the following page describes each goal as it pertains to the student and the mathematics curriculum. These six goals are an integral part of each standard and benchmark and are a necessary component of any comprehensive mathematics curriculum.

Setting goals and high expectations for all students in mathematics is essential to a mathematically literate society. The notion of what is “basic” in mathematics has expanded dramatically with the explosion of information and technology, and an increasingly global economy. Therefore, our curriculum must reflect a commitment to meet these new demands. The North Dakota Mathematics Content and Achievement Standards were designed to inform, assist, and advise all stakeholders in public education as we work together to prepare our students to meet the challenges of the twenty-first century.

(Chart will go here.)

## Standard 1: Number and Operation

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Kindergarten</p> <p>NUMBERS, NUMBER RELATIONSHIPS, AND NUMBER SYSTEMS</p> <ul style="list-style-type: none"> <li>• Count to 20</li> <li>• Count backward from 10 to 1</li>   <li>• Demonstrate one-to-one correspondence by counting up to 10 objects</li>   <li>• Identify ordinal numbers to order objects, 1<sup>st</sup> – 5<sup>th</sup></li>   <li>• Identify and write numerals to 10</li> </ul>	<p>Students correctly count forward and backward beyond 20.</p> <p>Students correctly count more than 10 objects.</p> <p>Students correctly identify ordinal numbers beyond 5<sup>th</sup>.</p> <p>Students correctly identify and write numerals beyond 10.</p>	<p>Students correctly count forward to 20 and backward from 10.</p> <p>Students correctly count 10 objects.</p> <p>Students correctly identify ordinal numbers 1<sup>st</sup> –5<sup>th</sup>.</p> <p>Students correctly identify and write numerals to 10.</p>	<p>Students count forward to 20 and backward from 10 with few errors.</p> <p>Students count up to 10 objects with few errors.</p> <p>Students correctly identify some of the ordinal numbers 1st-5<sup>th</sup>.</p> <p>Students identify and write numerals to 10 with few errors.</p>	<p>Students are unable to count forward to 20 and backward from 10.</p> <p>Students are unable to count 10 objects.</p> <p>Students are unable to identify ordinal numbers.</p> <p>Students identify and write numerals to 5 with few errors.</p>

Standard 1: Students understand and use basic and advanced concepts of number and number systems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Determine the relationship between two sets with 10 or fewer objects; i.e., less than, greater than, or equal to</li>   <li>• Use concrete materials to represent wholes and halves</li> </ul>	<p>Students correctly determine the relationship between two sets with more than 10 objects.</p> <p>Students use concrete materials to correctly represent common fractions.</p>	<p>Students correctly determine the relationship between two sets.</p> <p>Students use concrete materials to correctly represent wholes and halves.</p>	<p>Students have difficulty determining the relationship between two sets.</p> <p>Students use concrete materials to represent wholes and halves with few errors.</p>	<p>Students are unable to identify the relationship between two sets.</p> <p>Students incorrectly use concrete materials to represent wholes and halves.</p>
<p><b>OPERATIONS AND THEIR PROPERTIES</b></p> <ul style="list-style-type: none"> <li>• No expectations at this level</li> </ul>				
<p><b>COMPUTATIONAL FLUENCY AND ESTIMATION</b></p> <ul style="list-style-type: none"> <li>• No expectations at this level</li> </ul>				

## Standard 2: Geometry and Spatial Sense

Standard 2: Student understands and applies geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Kindergarten</p> <p><b>TWO- AND THREE-DIMENSIONAL SHAPES, GEOMETRIC PROPERTIES AND RELATIONSHIPS</b></p> <ul style="list-style-type: none"> <li>Identify and reproduce two-dimensional figures; i.e., circle, triangle, rectangle, and square</li> <li>Match a three-dimensional object with its stated name; i.e., cone, sphere, cube, cylinder (e.g., which of these is a cone?)</li> </ul> <p><b>COORDINATE GEOMETRY</b></p> <ul style="list-style-type: none"> <li>No expectations at this level</li> </ul> <p><b>TRANSFORMATION AND SYMMETRY</b></p> <ul style="list-style-type: none"> <li>No expectations at this level</li> </ul> <p><b>VISUALIZATION, SPATIAL REASONING, AND GEOMETRIC MODELING</b></p> <ul style="list-style-type: none"> <li>Identify position and direction; i.e., inside, outside, above, below, left and right</li> </ul>	<p>Students correctly identify and reproduce two-dimensional figures beyond grade level expectations.</p> <p>Students consistently match and name a three-dimensional object.</p> <p>Students correctly identify position and direction beyond grade level expectations.</p>	<p>Students correctly identify and reproduce two-dimensional figures.</p> <p>Students consistently match a three-dimensional object with its name.</p> <p>Students correctly identify position and direction.</p>	<p>Students identify and reproduce two-dimensional figures with some errors.</p> <p>Students inconsistently match a three-dimensional object with its name.</p> <p>Students correctly identify position and inconsistently identify direction.</p>	<p>Students are unable to identify and reproduce two-dimensional figures.</p> <p>Students are unable to match a three-dimensional object with its name.</p> <p>Students are unable to identify position and direction.</p>

### Standard 3: Data Analysis, Statistics, and Probability

Standard 3: Students use data collection and analysis techniques, statistical methods, and probability to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Kindergarten</p> <p><b>DATA COLLECTION, DISPLAY, AND INTERPRETATION</b></p> <ul style="list-style-type: none"> <li>• Use picture graphs as sources of information</li> </ul> <p><b>PROBABILITY</b></p> <ul style="list-style-type: none"> <li>• No expectations at this level</li> </ul> <p><b>STATISTICAL METHODS</b></p> <ul style="list-style-type: none"> <li>• No expectations at this level</li> </ul> <p><b>PREDICTIONS, DATA ANALYSIS AND INFERENCES</b></p> <ul style="list-style-type: none"> <li>• No expectations at this level</li> </ul>	<p>Students demonstrate comprehensive understanding of picture graphs as sources of information.</p>	<p>Students demonstrate substantial understanding of picture graphs as sources of information.</p>	<p>Students demonstrate a limited understanding of picture graphs as sources of information.</p>	<p>Students lack understanding of picture graphs as sources of information.</p>

## Standard 4: Measurement

Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Kindergarten</p> <p>MEASURABLE ATTRIBUTES, MEASUREMENT SYSTEMS AND UNITS</p> <ul style="list-style-type: none"> <li>• Name the days of the week in order</li>   <li>• Identify pictures to show knowledge of day and night</li>   <li>• Tell time to the hour using digital and analog clocks</li>   <li>• Order pictures first, next, last; based on time</li>   <li>• Compare and order objects according to their length or weight</li> </ul>	<p>Students consistently name the days of the week in order and use them to show knowledge of yesterday, today and tomorrow.</p> <p>Students identify events to show substantial knowledge of day and night and relate it to time.</p> <p>Students correctly tell time to the hour and beyond.</p> <p>Students accurately order events first, next, last; based on time.</p> <p>Students measure, then compare and correctly order objects according to their length or weight.</p>	<p>Students consistently name the days of the week in order.</p> <p>Students identify pictures to show substantial knowledge of day and night.</p> <p>Students correctly tell time to the hour.</p> <p>Students accurately order pictures first, next, last; based on time.</p> <p>Students compare and correctly order objects according to their length or weight.</p>	<p>Students inconsistently name the days of the week in order.</p> <p>Students identify pictures to show knowledge of day and night.</p> <p>Students tell time to the hour with few errors.</p> <p>Students order pictures first, next, last; based on time, with few errors.</p> <p>Students compare and order objects according to their length or weight, with few errors.</p>	<p>Students are unable to name the days of the week in order.</p> <p>Students show limited knowledge of day and night.</p> <p>Students are unable to tell time to the hour.</p> <p>Students order pictures first, next, last; based on time, with many errors.</p> <p>Students compare and attempt to order objects according to their length or weight.</p>

NORTH DAKOTA  
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Standard 4: Students use concepts and tools of measurement to describe and quantify the world				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<ul style="list-style-type: none"> <li>• Identify a penny, nickel, and dime and state its value</li> </ul>	<p>Students correctly identify all coins and state the appropriate value.</p>	<p>Students correctly identify a penny, nickel, and dime and state the appropriate value.</p>	<p>Students correctly identify a penny, nickel, or dime and/or state the appropriate value.</p>	<p>Students correctly identify a penny, nickel, or dime but are unable to state the appropriate value.</p>
<p>MEASUREMENT TOOLS, TECHNIQUES, AND FORMULAS</p> <ul style="list-style-type: none"> <li>• Measure length with non-standard units; e.g., paper clips, cubes</li> </ul>	<p>Students accurately measure length with non-standard and standard units.</p>	<p>Students accurately measure length with non-standard units.</p>	<p>Students inconsistently measure length with non-standard units.</p>	<p>Students are unable to measure length with non-standard units.</p>

## Standard 5: Algebra, Functions, and Patterns

Standard 5: Students use algebraic concepts, functions, patterns, and relationships to solve problems				
BENCHMARK EXPECTATION	PROFICIENCY DESCRIPTOR			
	ADVANCED PROFICIENT	PROFICIENT	PARTIALLY PROFICIENT	NOVICE
<p>Kindergarten</p> <p><b>PATTERNS, RELATIONS, AND FUNCTIONS</b></p> <ul style="list-style-type: none"> <li>Identify, sort, and classify objects by their common attribute</li>   <li>Recognize, extend, and describe simple patterns</li> </ul> <p><b>NUMERIC AND ALGEBRAIC REPRESENTATIONS</b></p> <ul style="list-style-type: none"> <li>No expectations at this level</li> </ul> <p><b>MATHEMATICAL MODELING</b></p> <ul style="list-style-type: none"> <li>Use tools and strategies, (e.g., manipulatives) to model problems</li> </ul> <p><b>RATES OF CHANGE</b></p> <ul style="list-style-type: none"> <li>No expectations at this level</li> </ul>	<p>Students accurately identify, sort, and classify objects by multiple attributes.</p> <p>Students correctly recognize, extend, and describe various patterns.</p> <p>Students accurately use tools and strategies to model and solve problems.</p>	<p>Students accurately identify, sort, and classify objects by their common attribute.</p> <p>Students correctly recognize, extend, and describe simple patterns.</p> <p>Students accurately use tools and strategies to model problems.</p>	<p>Students inconsistently identify, sort, and classify objects by their common attribute.</p> <p>Students demonstrate beginning knowledge to recognize, extend, and describe simple patterns.</p> <p>Students have beginning understanding when using tools and strategies to model problems.</p>	<p>Students lack understanding to identify, sort, and classify objects by their common attribute.</p> <p>Students are unable to recognize, extend, and describe simple patterns.</p> <p>Students lack the knowledge when to use tools and strategies to model problems.</p>