

# **North Dakota Standards and Benchmarks**

## **Achievement Standards**

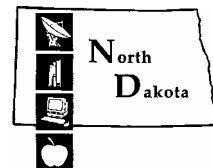
### ***Mathematics***

Version 3.0

### **Grade 12**

**2000-2001**

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## Standard 1: Numbers and Operations

*Students understand and use basic and advanced concepts of number and number systems.*

### Benchmark

12.1.1 *Know and use the real number system, its subsets and properties.*

**Level 4** The student articulates the relationships among the subsets (natural, whole, integers, rational, and irrational) of the real numbers and properties (commutative, distributive, associative, closure, identity, and inverse) which apply to each subset.

**Level 3** The student articulates all the subsets (natural, whole, integers, rational and irrational numbers) and all the properties (commutative, distributive, associative, closure, identity, and inverse) of the real number system.

**Level 2** The student articulates 3 or 4 of the subsets (natural, whole, integers, rational, and irrational) and 3 - 5 of the properties (commutative, distributive, associative, closure, identity, and inverse) of the real number system.

**Level 1** The student articulates 1 or 2 of the subsets (natural, whole, integers, rational, and irrational) and 1 or 2 of the properties (commutative, distributive, associative, closure, identity, and inverse) of the real number system.

12.1.2 *Identify complex numbers and understand their relevance in solving equations.*

**Level 4** The student recognizes, by examining a polynomial equation or graph, if an equation has complex roots (without solving the equation), accurately solves the polynomial equation, and explains the relevance of the roots.

**Level 3** The student recognizes, by examining a polynomial equation or graph, if an equation has complex roots (without solving the equation) and accurately solves the polynomial equation.

**Level 2** The student accurately finds and identifies complex roots of polynomial equations.

**Level 1** The student accurately identifies complex numbers.

12.1.3 *Use basic set relations and operations with appropriate notation*

**Level 4** The student accurately finds union, intersection, subsets, and elements of sets and uses appropriate notation with each. The student demonstrates real world applications of at least 3 of the following concepts: union, intersection, subsets, and elements of sets.

**Level 3** The student accurately finds union, intersection, subsets, and elements of sets and uses appropriate notation with each.

**Level 2** The student accurately finds 3 of the following: union, intersection, subsets, or elements of sets.

**Level 1** The student accurately finds 1 or 2 of the following: union, intersection, subsets, or elements of sets.

12.1.4	<p><i>Understand the meaning of operations and how they relate to each other.</i></p>
	<p><b>Level 4</b> The student accurately performs basic operations using both numbers and variables with scalars, complex numbers, vectors, and matrices.  <b>Level 3</b> The student accurately performs basic operations with scalars, complex numbers, vectors, and matrices.  <b>Level 2</b> The student attempts to perform basic operations with scalars, complex numbers, vectors, and matrices, but makes minor mistakes in calculations.  <b>Level 1</b> The student unsuccessfully attempts to perform basic operations with scalars, complex numbers, vectors, and matrices.</p>
12.1.5	<p><i>Apply advanced estimation skills and appropriate computational methods to attain reasonable solutions.</i></p>
	<p><b>Level 4</b> The student obtains a reasonable estimate (<math>\pm 10\%</math> of actual value) based on a valid strategy and provides an explanation of possible errors.  <b>Level 3</b> The student finds a reasonable estimate (<math>\pm 10\%</math> of actual value) based on a valid strategy.  <b>Level 2</b> The student finds an estimate but has minor computational error(s) or has used an invalid strategy.  <b>Level 1</b> The student makes an estimate, shows no computation, and has error within <math>\pm 10 - 20\%</math> of actual value.</p>
12.1.6	<p><i>Understand the properties and basic theorems of roots, exponents, and logarithms.</i></p>
	<p><b>Level 4</b> The student effectively manipulates expressions and equations that involve integral, fractional, and variable exponents, including equations that require the use of logarithms.  <b>Level 3</b> The student effectively manipulates expressions and equations that involve integral, fractional, and variable exponents.  <b>Level 2</b> The student effectively manipulates expressions or equations with integral and fractional exponents.  <b>Level 1</b> The student attempts to manipulate expressions or equations with integral or fractional exponents</p>

## Standard 2: Geometry and Spatial Sense

*Students understand and apply geometric concepts and spatial relationships to represent and solve problems in mathematical and nonmathematical situations.*

### Benchmark

- 12.2.1 *Understand and apply the properties of two- and three-dimensional figures.*
- Level 4** The student accurately represents a three-dimensional object using a two-dimensional **scale** drawing.  
**Level 3** The student accurately represents a three-dimensional object using a two-dimensional drawing.  
**Level 2** The student represents a three-dimensional object using a two-dimensional drawing with minor errors, for example, a net that does not fold accurately or inaccurate angle(s) in a perspective drawing.  
**Level 1** The student attempts to draw a two-dimensional figure of a three-dimensional object.
- 12.2.2 *Construct basic geometric figures using appropriate tools.*
- Level 4** The student accurately constructs basic geometric figures using appropriate tools and explains why the process creates the desired result (may make reference to theorems or postulates).  
**Level 3** The student accurately constructs basic geometric **figures** using appropriate tools.  
**Level 2** The student accurately performs basic geometric **constructions** using appropriate tools.  
**Level 1** The student attempts to perform basic geometric constructions.
- 12.2.3 *Understand the concepts of congruence, similarity, and symmetry.*
- Level 4** The student correctly identifies congruent and similar figures using appropriate terminology and defends reasoning using definitions, postulates or theorems. The student identifies any type of symmetry present.  
**Level 3** The student correctly identifies congruent and similar figures using appropriate terminology. The student identifies any type of symmetry present.  
**Level 2** The student identifies congruent and/or similar figures using some appropriate terminology.  
**Level 1** The student identifies any correct relationship between figures but does not use appropriate terminology.
- 12.2.4 *Apply transformations to basic shapes.*
- Level 4** The student performs and recognizes translations, reflections, rotations, and dilations AND the student describes the transformations using appropriate terminology, such as but not limited to, the units with respect to the coordinate system, degrees of clockwise or counterclockwise rotations, percent of dilation, isometry.  
**Level 3** The student performs and recognizes translations, reflections, rotations, and dilations.  
**Level 2** The student performs and recognizes 3 of the following transformations: translations, reflections, rotations, and dilations.  
**Level 1** The student performs and recognizes 1 or 2 of the following transformations: translations, reflections, rotations, and dilations.

12.2.5	<p><i>Apply the Pythagorean Theorem to basic shapes.</i></p> <p><b>Level 4</b> The student applies the Pythagorean Theorem accurately in appropriate situations and explains why the Pythagorean Theorem was used.  <b>Level 3</b> The student applies the Pythagorean Theorem accurately in appropriate situations.  <b>Level 2</b> The student applies the Pythagorean Theorem but has an incorrect answer.  <b>Level 1</b> The student is unable to apply the Pythagorean Theorem. OR The student states the Pythagorean Theorem incorrectly, such as <math>a + b = c</math>.</p>
12.2.6	<p><i>Apply basic trigonometric ratios to solve real-world problems.</i></p> <p><b>Level 4</b> The student identifies the given information and chooses the appropriate trigonometric ratio/function to accurately solve for the unknown AND provides either an appropriate explanation or checks using a second ratio.  <b>Level 3</b> The student identifies the given information and chooses an appropriate trigonometric ratio/function to accurately solve for the unknown.  <b>Level 2</b> The student identifies the given information and attempts to use a trig ratio/function to find the unknown, minor errors in computation or sets up wrong ratio/function.  <b>Level 1</b> The student attempts to find the unknown but is unable to apply a trigonometric ratio/function.</p>
12.2.7	<p><i>Apply measurements and formulas in computations of perimeter, area, and volume.</i></p> <p><b>Level 4</b> The student applies measurements and formulas to correctly calculate perimeter, area, and volume. Correct labels are used. The student discusses possible errors and/or other factors that could effect outcomes of calculations.  <b>Level 3</b> The student applies measurement and formulas correctly to calculate perimeter, area, and volume.  <b>Level 2</b> The student uses formulas correctly, but makes minor error(s).  <b>Level 1</b> The student uses wrong formula(s) or uses formula(s) incorrectly.</p>
12.2.8	<p><i>Generate geometric conjectures inductively and validate them deductively.</i></p> <p><b>Level 4</b> The student formulates a conjecture that is true, can prove it deductively, and is able to make additional related conjecture(s).  <b>Level 3</b> The student formulates a conjecture that is true and can prove it deductively.  <b>Level 2</b> The student formulates a conjecture that appears to be true and attempts to prove it  <b>Level 1</b> The student formulates a conjecture that appears to be true with no attempt to prove it.</p>
12.2.9	<p><i>Apply geometric properties to a coordinate system.</i></p> <p><b>Level 4</b> The student accurately applies geometric properties to a coordinate system. The student clearly presents complete support of the application from at least two different perspectives.  <b>Level 3</b> The student accurately applies geometric properties to a coordinate system. The student presents complete support of the application.  <b>Level 2</b> The student attempts to apply geometric properties to a coordinate system. The student presents inaccurate or incomplete support of the application.  <b>Level 1</b> The student attempts to apply geometric properties to a coordinate system. The student is not able to support the application.</p>

### Standard 3: Data Analysis, Statistics, and Probability

*Students use data collection and analysis techniques, statistical methods, and probability to solve problems.*

#### **Benchmark**

12.3.1 *Design, implement, and present statistical studies*

**Level 4** The student successfully completes the experiment by performing the necessary trials that would make for a statistically sound experiment. The student records and interprets the data correctly, but also reflects on the appropriateness of the results. The solution process is complete and accurate.

**Level 3** The student performs the necessary trials to a predetermined experiment. The data is accurately recorded and interpreted in relationship to the trials for the experiment.

**Level 2** The student can do any two of the following requirements for an experiment: (1) perform the correct number of trials, (2) record the information correctly in relation to the trials for the experiment, (3) accurately interpret the information from the trials.

**Level 1** The student can do any one of the following requirements for an experiment: (1) perform the correct number of trials, (2) record the information correctly in relation to the trials for the experiment, (3) accurately interpret the information from the trials.

12.3.2 *Sample data and understand the role of sampling in data analysis.*

**Level 4** Student demonstrates the ability to choose a random sample from a given population, uses statistical data to correctly determine population parameters, and explains appropriate sampling techniques such as, but not limited to, sample size and application to central limit theorem.

**Level 3** Student demonstrates the ability to choose a random sample from a given population and uses statistical data to correctly determine population parameters.

**Level 2** Student attempts to choose a random sample but uses faulty method and/or makes errors in use of appropriate method or student chooses a random sample but is unable to use statistical data to correctly determine population parameters.

**Level 1** Student chooses a non-random sample.

12.3.3 *Use counting strategies.*

**Level 4** The student recognizes the need to use a counting strategy such as factorial, permutation, combination, or counting principle; and obtains an accurate result using more than one method. OR The student recognizes the need to use a counting strategy such as factorial, permutation, combination, or counting principle; accurately applies the correct concept; and is able to explain using correct terminology why that concept is appropriate.

**Level 3** The student recognizes the need to use a counting strategy such as factorial, permutation, combination, or counting principle; and obtains an accurate result using one method.

**Level 2** The student recognizes the need to use a counting strategy such as factorial, permutation, combination, or counting principle; but is unable to accurately apply the correct method or uses correct method with incorrect calculations.

**Level 1** Student lacks the ability to use counting strategies without assistance.

12.3.4	<p><i>Calculate theoretical and experimental probabilities.</i></p> <p><b>Level 4</b> The student calculates correctly, compares, and contrasts experimental and theoretical probability.  <b>Level 3</b> The student calculates correctly both experimental and theoretical probability.  <b>Level 2</b> The student calculates correctly an experimental or theoretical probability.  <b>Level 1</b> The student attempts to find a probability but makes an error in computation and/or uses incorrect formula.</p>
12.3.5	<p><i>Calculate and interpret measures of central tendency and variance.</i></p> <p><b>Level 4</b> The student calculates accurately mean, median, mode, and at least 2 measures of dispersion (range, standard deviation, variance, and maximum / minimum).  OR The student calculates accurately at least 2 measures of central tendency (mean, median, or mode) and 1 measure of dispersion (range, standard deviation, variance, or maximum /minimum). The student accurately interprets the statistical results.  <b>Level 3</b> The student calculates accurately at least 2 measures of central tendency (mean, median, and mode) and 1 measure of dispersion (range, standard deviation, variance, and maximum / minimum).  <b>Level 2</b> The student calculates accurately any 2 statistical measures (mean, median, mode, range, standard deviation, variance, and maximum / minimum).  <b>Level 1</b> The student calculates accurately any 1 statistical measure (mean, median, mode, range, standard deviation, variance, or maximum / minimum).</p>
12.3.6	<p><i>Use regression techniques to determine and interpret the curve of best fit.</i></p> <p><b>Level 4</b> The student uses regression techniques to correctly determine the curve of best fit, writes an accurate equation, and analyzes how well the actual data fits the model.  <b>Level 3</b> The student uses regression techniques to correctly determine the curve of best fit and writes an accurate equation.  <b>Level 2</b> The student attempts to draw the curve of best fit and finds equation but has errors in coefficients.  <b>Level 1</b> Student attempts to draw the curve of best fit but is unable to find an equation.</p>
12.3.7	<p><i>Draw inferences and predict outcomes from data expressed in a variety of ways.</i></p> <p><b>Level 4</b> The student makes a reasonable prediction based on given information and discusses or uses a second method to verify reasonableness.  <b>Level 3</b> Student makes a reasonable prediction based on given information, such as but not limited to: graphs, equations, and sample data.  <b>Level 2</b> The student makes a prediction but makes an error in computation and/or theory application.  <b>Level 1</b> The student makes a reasonable prediction with no basis (appears to be a guess).</p>

## Standard 4: Measurement

*Students use concepts and tools of measurement to describe and quantify the world.*

### Benchmark

- 12.4.1 *Understand attributes, units, and systems of measurement.*
- Level 4** The student chooses an appropriate unit of measure and explains why that unit of measurement is appropriate (may discuss precision and/or accuracy).  
**Level 3** The student chooses an appropriate unit of measure.  
**Level 2** The student chooses the correct system of measurement but makes poor choice of units.  
**Level 1** The student uses the wrong system of measurement.
- 12.4.2 *Apply a variety of techniques, tools, and formulas to determine measurements.*
- Level 4** The student accurately finds a measurement using more than 1 technique, tool, or formula and accurately explains the relationships between the methods (e.g. trigonometry or similar figures).  
**Level 3** The student accurately finds a measurement using more than 1 technique, tool, or formula (e.g. trigonometry or similar figures).  
**Level 2** The student demonstrates a knowledge of more than 1 technique, tool, or formula to find a measurement but makes errors in calculations and/or techniques (e.g. trigonometry or similar figures).  
**Level 1** The student finds a measurement using only 1 technique, tool, or formula (e.g. trigonometry or similar figures).
- 12.4.3 *Measure physical quantities and determine measurement error.*
- Level 4** The student measures accurately (within 2%) and discusses error in measurement and/or compounded error in calculation based on measurement.  
**Level 3** The student measures accurately within 2% of correct measurement.  
**Level 2** The student gives a measurement within 5% of correct measurement.  
**Level 1** The student makes error in measurement that is greater than 5% of correct measurement.
- 12.4.4
- Level 4** The student explains or demonstrates clearly an appropriate estimation strategy and either provides insight into factor(s) that might affect the estimate or provides multiple approaches to the problem.  
**Level 3** The student explains or demonstrates clearly an appropriate estimation strategy.  
**Level 2** The student explains or demonstrates an appropriate, but incomplete estimation strategy.  
**Level 1** The student has an estimation that is based on a faulty strategy

## Standard 5: Algebra, Functions, and Patterns

*Students use algebraic concepts, functions, patterns, and relationships to solve problems.*

### Benchmark

12.5.1 *Use algebraic procedures to manipulate mathematical expressions.*

**Level 4** The student demonstrates the ability to manipulate algebraic expressions and identifies the specific properties used in each step of the process.

**Level 3** The student demonstrates the ability to manipulate algebraic expressions using various properties such as but not limited to commutative, associative, and distributive.

**Level 2** The student uses algebraic procedures to manipulate expressions with minor errors.

**Level 1** The student lacks the ability to accurately manipulate algebraic expressions without assistance.

12.5.2 *Solve equations, inequalities, and systems.*

**Level 4** The student accurately solves equations, inequalities, and systems of equations. The student also recognizes inappropriate solutions and/or places restrictions on variables where necessary.

**Level 3** The student accurately solves equations, inequalities, and systems of equations.

**Level 2** The student demonstrates the ability to solve equations, inequalities, and systems of equations but makes minor mistakes in calculations.

**Level 1** The student lacks the ability to accurately solve equations, inequalities, and systems of equations without assistance.

12.5.3 *Represent and describe relations algebraically, numerically, and graphically.*

**Level 4** The student accurately represents relations algebraically, numerically, and graphically. The student also identifies advanced descriptors.

**Level 3** The student accurately represents relations algebraically, numerically, and graphically and the student identifies basic descriptors.

**Level 2** The student accurately represents relations in at least two of the following ways: algebraically, numerically, and graphically.

**Level 1** The student accurately represents relations in one of the following ways: algebraically, numerically, or graphically.

12.5.4 *Create, manipulate, and apply matrices to real-life situations.*

**Level 4** The student correctly constructs a matrix, including correct dimensions, entries, brackets, and row/column labels. The student performs basic operations between matrices. The student recognizes the appropriate matrix operation needed to model a real-life situation.

**Level 3** The student correctly constructs a matrix, including correct dimensions, entries, brackets, and row/column labels. The student performs basic operations between matrices.

**Level 2** The student constructs a matrix with any three of the following features correct: matrix has correct dimensions, entries are correct, matrix is enclosed in brackets, and rows/columns are correctly labeled.

**Level 1** The student constructs a matrix with any two of the following features correct: matrix has correct dimensions, entries are correct, matrix is enclosed in brackets, and rows/columns are correctly labeled.

12.5.5	<p><i>Develop and analyze a variety of algorithms.</i></p> <p><b>Level 4</b> The student chooses an appropriate algorithm, uses it correctly, and logically describes why the process produces the desired outcome. The student also develops alternative algorithms or applies them to real-life situations.  <b>Level 3</b> The student chooses an appropriate algorithm, uses it correctly, and logically describes why the process produces the desired outcome.  <b>Level 2</b> The student chooses an appropriate algorithm and uses it correctly.  <b>Level 1</b> The student chooses an incorrect algorithm or makes errors while using a correct algorithm.</p>
12.5.6	<p><i>Understand and apply the process of recursion.</i></p> <p><b>Level 4</b> The student continues a sequence and develops the recursive formula for the <math>n</math>th term of the sequence. The student also verifies the recursive pattern by mathematical induction.  <b>Level 3</b> The student continues a sequence and develops the recursive formula for the <math>n</math>th term of the sequence.  <b>Level 2</b> The student continues a sequence using recursion.  <b>Level 1</b> The student is unable to find the next term of the sequence without assistance.</p>
12.5.7	<p><i>Use patterns and functions to model problems.</i></p> <p><b>Level 4</b> The student accurately determines a pattern or function that models a real-world situation. The student also recognizes inappropriate solutions and/or places restrictions on variables where necessary.  <b>Level 3</b> The student accurately determines a pattern or function that models a real-world situation.  <b>Level 2</b> The student models a real-world situation with a pattern or function that contains minor errors.  <b>Level 1</b> The student requires assistance to develop a pattern or function that models a real-world situation.</p>
12.5.8*	<p><i>Understand the basic ideas about convergence, limit of functions, and infinite series.</i></p> <p><b>Level 4</b> The student specifies which series converge or diverge and correctly identifies the limit of a series, where appropriate. The student recognizes real world problems that involve infinite series and finds the limit.  <b>Level 3</b> The student specifies which series converge or diverge and correctly identifies the limit of a series, where appropriate.  <b>Level 2</b> The student recognizes the difference between convergence and divergence but is unable to identify the limit of a series.  <b>Level 1</b> The student needs assistance to recognize the difference between convergence and divergence.</p> <p>* is used to distinguish material that is necessary for college-intending students but it is not necessarily required for all graduating high school students.</p>