

## Meet the Fact family.

The question is not **if** students should learn the math facts, **but how?**

## Our goal

- ...every student learns the basic facts
  - efficiently,
  - to gain fluency with their use,
  - to retain that fluency over time.

Which teaching methods are most **efficient** and **effective**?

## Research supports work with strategies & concepts

1. Early emphasis on problem solving rather than memorizing facts.
2. De-emphasis of rote work.
3. Gradual introduction of facts.
4. Ongoing practice.
5. Appropriate assessment.
6. Multi-year approach.
7. Facts are not gate-keepers

| Grade    | Addition  | Subtraction   | Multiplication  | Division  |
|----------|---|---|---|---|
| <b>K</b> | <ul style="list-style-type: none"> <li>• invented strategies</li> </ul>                               | <ul style="list-style-type: none"> <li>• invented strategies</li> </ul>                               |   |   |
| <b>1</b> | <ul style="list-style-type: none"> <li>• strategies</li> </ul>  | <ul style="list-style-type: none"> <li>• strategies</li> </ul>  |   |   |
| <b>2</b> | <ul style="list-style-type: none"> <li>• strategies</li> <li>• practice leading to fluency</li> </ul> | <ul style="list-style-type: none"> <li>• strategies</li> <li>• practice leading to fluency</li> </ul> |   |   |
| <b>3</b> | <ul style="list-style-type: none"> <li>• review and practice</li> </ul>                               | <ul style="list-style-type: none"> <li>• review and practice</li> </ul>                               | <ul style="list-style-type: none"> <li>• strategies</li> <li>• practice leading to fluency</li> </ul> |   |
| <b>4</b> | <ul style="list-style-type: none"> <li>• assessment and remediation as required</li> </ul>            | <ul style="list-style-type: none"> <li>• assessment and remediation as required</li> </ul>            | <ul style="list-style-type: none"> <li>• review and practice</li> </ul>                               | <ul style="list-style-type: none"> <li>• strategies</li> <li>• practice leading to fluency</li> </ul> |
| <b>5</b> | <ul style="list-style-type: none"> <li>• assessment and remediation as required</li> </ul>            | <ul style="list-style-type: none"> <li>• assessment and remediation as required</li> </ul>            | <ul style="list-style-type: none"> <li>• review and practice</li> </ul>                               | <ul style="list-style-type: none"> <li>• review and practice</li> </ul>                               |

## Expectations by Grade Level (consistent with NCTM)

- Kindergarten – Manipulatives and inventive strategies for + and – .
- 1<sup>st</sup> – Students can solve basic + and – problems using some strategy. Begin on X problems. Fluency is not emphasized.
- 2<sup>nd</sup> – Efficient strategies for + and - are emphasized. Fluency by end of year. Continue with X facts.

## Expectations by Grade Level (consistent with NCTM)

- 3<sup>rd</sup> – Review + and – facts and develop strategies for X facts. Fluency by end of year.
- 4<sup>th</sup> – Review X facts and develop strategies for ÷ facts. Fluency by end of year.
- 5<sup>th</sup> – Review X and ÷ facts. Fluency by end of year for all facts.

## Strategies for Learning the Facts

### Addition & Subtraction Facts

## Counting All

$7 + 8$

$7 + 8 = 15$

## Counting On - Addition

$8 + 3$

$8 + 3 = 11$

## Counting On

Dominoes & dice

$$8 + 3 = 11$$

Even if counters are not used, finger gestures can keep track of how many more to count.



## Counting On - Subtraction

Dominoes & dice

$$11 - 8 =$$

11 cars

9      10      11

Count on from 8

$$11 - 8 = 3$$

## Counting Back for Subtraction

Dominoes & dice

- Not easy
- Provide practice

### Tools to practice with



Hundreds Chart



Calendar

Practice counting backwards

11 - 10 - 9 - 8 - ...

6 - 5 - 4 - 3 - ...

22 - 21 - 20 - 19 - ...

## Doubles

$1+1=2$     $2+2=4$     $3+3=6$     $4+4=8$

$5+5=10$



$6+6=12$



$7+7=14$

$8+8=16$

$9+9=18$

## Near Doubles

**1 More** from known fact

$5+4=?$

$4+4=8$

so  $5+4$  is **one more** than 8

Therefore,

$5+4=9$

## Near Doubles

### **1 Less** from known fact

$6+5=?$

$6+6=12$

so  $6+5$  is **one less** than 12

Therefore,

$6+5=11$

## Reasoning Practice

worksheet

|       | Double Fact   | 1 more or           | 1 less              |
|-------|---|---------------------|---------------------|
| $3+4$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ | $\underline{\quad}$ | $\underline{\quad}$ |
| $6+7$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ | $\underline{\quad}$ | $\underline{\quad}$ |
| $8+9$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ | $\underline{\quad}$ | $\underline{\quad}$ |
| $5+6$ | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ | $\underline{\quad}$ | $\underline{\quad}$ |

## Practice



How much is  $6+6$  ?

Add 1 more to one of the groups.

Now, we have  $6+7$  .

How many do we have altogether?

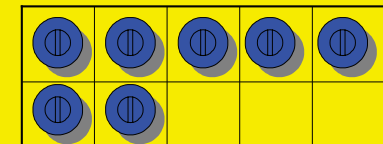
1 more than 12 is 13.

So,  $6+7=13$

## Making a 10

Facts with sum of 10 are easier.

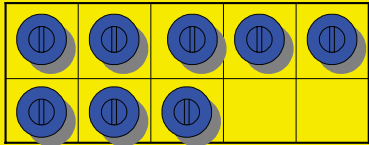
$2+8=10$     $7+3=10$     $6+4=10$



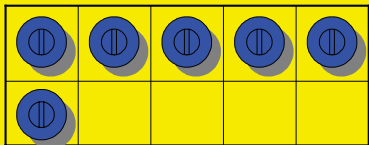
$7+3=10$

## Making a 10

Facts with sum of 10 are easier.



$$8 + 2 = 10$$



$$6 + 4 = 10$$

## Making a 10 Practice

worksheet

$$\underline{7} + \underline{\quad} = \underline{10} \quad \& \quad \underline{\quad} + \underline{7} = \underline{10}$$

$$\underline{6} + \underline{\quad} = \underline{10} \quad \& \quad \underline{\quad} + \underline{6} = \underline{10}$$

so  $\underline{\quad} - \underline{\quad} = \underline{\quad}$  and  $\underline{\quad} - \underline{\quad} = \underline{\quad}$

## Compensation

Increasing 1 addend while decreasing the other by the same amount

Using counters:

$$8 \text{ and } 4 \quad 8 + 2 = 10$$

$$4 - 2 = \underline{2}$$

$$\text{So, } 8 + 4 = 12$$

## Compensation Practice

worksheet

$$9 + 5 \quad \text{is the same as} \quad 10 + \underline{\quad}$$

$$5 + 7 \quad \text{is the same as} \quad 10 + \underline{\quad}$$

$$8 + 7 \quad \text{is the same as} \quad 10 + \underline{\quad}$$

$$9 + 6 \quad \text{is the same as} \quad 10 + \underline{\quad}$$

## All Combinations of a Sum

Colored discs 8's

$$\underline{\quad} + \underline{\quad} = 8$$

$$4 + 4 = 8$$

$$5 + 3 = 8$$

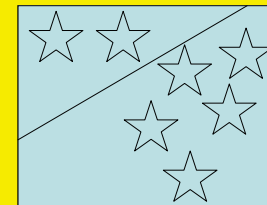
$$2 + 6 = 8$$

$$7 + 1 = 8$$

$$8 + 0 = 8$$

Worksheet 9's

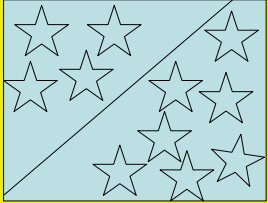
## Fact Families



## Practice

worksheet

file folders with stickers



Addition

$$\begin{array}{r} \underline{\quad} + \underline{\quad} = \underline{\quad} \\ \underline{\quad} + \underline{\quad} = \underline{\quad} \end{array}$$

## Thinking Strategies for X

- Repeated addition
- Skip counting
- Splitting the product into known parts
- Facts of five
- Patterns
- Fact families

## Thinking Strategies for ÷

- Work with fact families

## Purpose of Strategies

Purpose is to give students:

- ways of learning the facts
- confidence to think problems through
- opportunity to choose the strategies that work best for them
- opportunity to invent their own

## Questions

Thank you for attending.

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### Fact Family Practice Worksheet

Near Doubles Reasoning

|       | Double Fact   | 1 more          | or | 1 less |
|-------|---|-----------------|----|--------|
| 3 + 4 | $\underline{3} + \underline{3} = \underline{6}$             | $\underline{X}$ |    |        |
| 6 + 7 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |
| 8 + 9 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |
| 5 + 6 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |
| 8 + 7 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |
| 7 + 6 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |
| 5 + 4 | $\underline{\quad} + \underline{\quad} = \underline{\quad}$ |                 |    |        |

Making a 10



|  |    |   |     |   |
|--|----|---|-----|---|
| $\underline{7} + \underline{3} = \underline{10}$     | so | $\underline{10} - \underline{7} = \underline{3}$            | and | $\underline{10} - \underline{3} = \underline{7}$            |
| $\underline{6} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{4} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{8} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{9} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{5} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{3} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |
| $\underline{2} + \underline{\quad} = \underline{10}$ | so | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ | and | $\underline{\quad} - \underline{\quad} = \underline{\quad}$ |

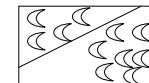
Compensation Practice

|                      |                          |
|----------------------|--------------------------|
| 9 + 5 is the same as | 10 + $\underline{\quad}$ |
| 5 + 7 is the same as | 10 + $\underline{\quad}$ |
| 8 + 7 is the same as | 10 + $\underline{\quad}$ |
| 9 + 6 is the same as | 10 + $\underline{\quad}$ |
| 8 + 5 is the same as | 10 + $\underline{\quad}$ |
| 9 + 7 is the same as | 10 + $\underline{\quad}$ |

Combinations of a sum

|   |   |
|---|---|
| $\underline{\quad} + \underline{\quad} = 9$ | $\underline{\quad} + \underline{\quad} = 9$ |
| $\underline{\quad} + \underline{\quad} = 9$ | $\underline{\quad} + \underline{\quad} = 9$ |
| $\underline{\quad} + \underline{\quad} = 9$ |   |

Fact Families



Addition

$$\begin{array}{r} \underline{\quad} + \underline{\quad} = \underline{\quad} \\ \underline{\quad} + \underline{\quad} = \underline{\quad} \end{array}$$

Subtraction

$$\begin{array}{r} \underline{\quad} - \underline{\quad} = \underline{\quad} \\ \underline{\quad} - \underline{\quad} = \underline{\quad} \end{array}$$